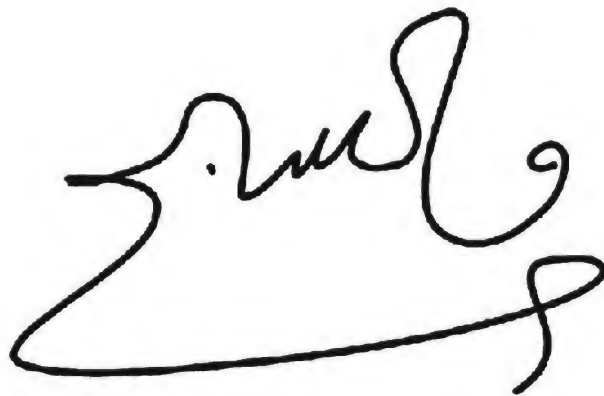


Example & Solution of Problem in Open Channel Hydraulics

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by

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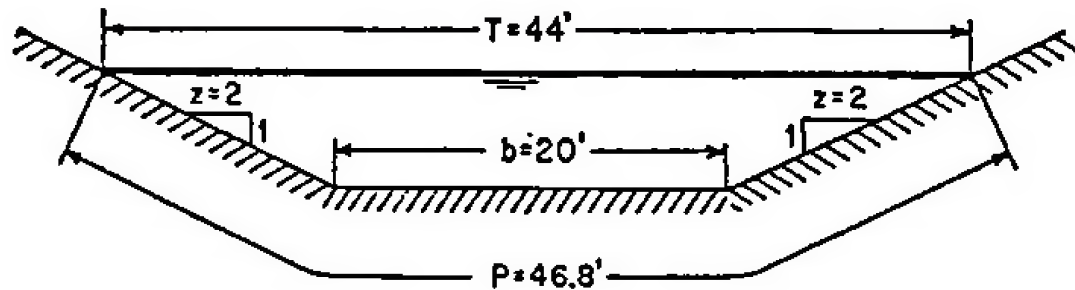
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Chapter one

Example 1: Compute the hydraulic radius, hydraulic depth and section factor of the trapezoidal channel section shown in figure below. The depth of flow is 6 ft.



Solution:

$$P = 20 + 2 \times 6 \times (5)^{1/2} = 46.8 \text{ ft};$$

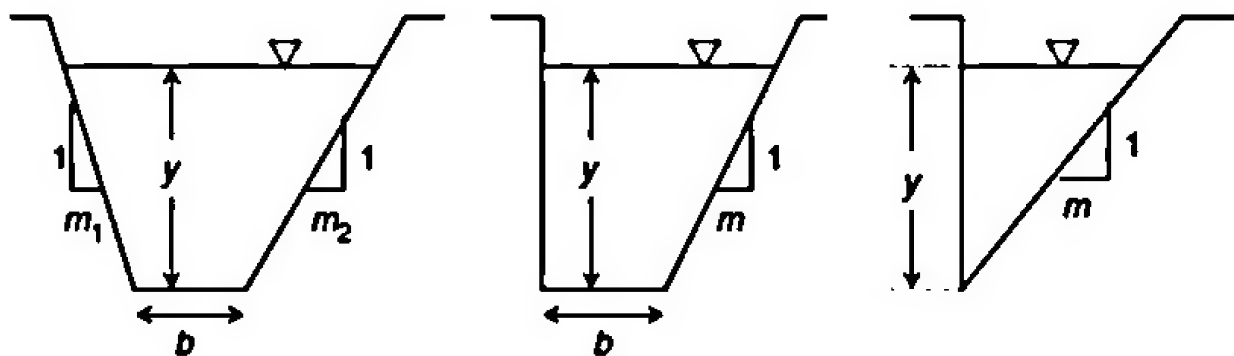
$$A = 0.5 \times (20 + 44) \times 6 = 192 \text{ ft}^2;$$

$$R = A / P = 192 / 46.8 = 4.1 \text{ ft};$$

$$D = 192 / 44 = 4.37 \text{ ft; and}$$

$$Z = 192 \times (4.37)^{0.5} = 401 \text{ ft}^{2.5}$$

Example 2: Derive expressions for the flow area A , wetted perimeter P , top width T , hydraulic radius R , and hydraulic depth D , in terms of flow depth y , for the channel sections shown in figure below.



Chapter Two

Example 2.1: Water at 10°C flows in a 6-m-wide rectangular channel at a depth of 0.55 m and a flow rate of 12 m³/s. Determine

(a) The critical depth, (b) Whether the flow is subcritical or supercritical, and (c) alternate depth.

Solution:

Assumptions The flow is uniform and thus the specific energy is constant.

Analysis (a) The critical depth is calculated to be $y_c = \left(\frac{\dot{V}^2}{gb^2} \right)^{1/3} = \left(\frac{(12 \text{ m}^3/\text{s})^2}{(9.81 \text{ m/s}^2)(6 \text{ m})^2} \right)^{1/3} = 0.742 \text{ m}$

(b) The average flow velocity and the Froude number are

$$V = \frac{\dot{V}}{by} = \frac{12 \text{ m}^3/\text{s}}{(6 \text{ m})(0.55 \text{ m})} = 3.636 \text{ m/s} \quad \text{and} \quad Fr_1 = \frac{V}{\sqrt{gy}} = \frac{3.636 \text{ m/s}}{\sqrt{(9.81 \text{ m/s}^2)(0.55 \text{ m})}} = 1.565, \text{ which is greater than 1.}$$

Therefore, the flow is **supercritical**.

(c) Specific energy for this flow is

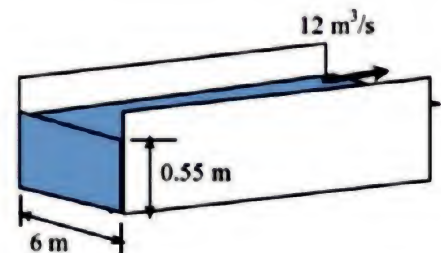
$$E_{s1} = y_1 + \frac{\dot{V}^2}{2gb^2y_1^2} = (0.55 \text{ m}) + \frac{(12 \text{ m}^3/\text{s})^2}{2(9.81 \text{ m/s}^2)(6 \text{ m})^2(0.55 \text{ m})^2} = 1.224 \text{ m}$$

Then the alternate depth is determined from $E_{s1} = E_{s2}$,

$$E_{s2} = y_2 + \frac{\dot{V}^2}{2gb^2y_2^2} \rightarrow 1.224 \text{ m} = y_2 + \frac{(12 \text{ m}^3/\text{s})^2}{2(9.81 \text{ m/s}^2)(6 \text{ m})^2y_2^2}$$

The alternate depth is calculated to be $y_2 = 1.03 \text{ m}$ which is the subcritical depth for the same value of specific energy.

Discussion The depths 0.55 m and 1.03 are alternate depths for the given discharge and specific energy. The flow conditions determine which one is observed.



Example 2.2: Water flows at a depth of 0.8 ft with an average velocity of 14 ft/s in a wide rectangular channel. Determine

(a) The Froude number, (b) The critical depth, (c) whether the flow is subcritical or supercritical, and (d) what would your response be if the flow depth were 0.2 ft?

Solution:

Assumptions The flow is uniform and thus the specific energy is constant.

Analysis (a) The Froude number is $Fr = \frac{V}{\sqrt{gy}} = \frac{14 \text{ ft/s}}{\sqrt{(32.2 \text{ ft/s}^2)(0.8 \text{ ft})}} = 2.76$

(b) The critical depth is calculated to be $y_c = \left(\frac{\dot{V}^2}{gb^2} \right)^{1/3} = \left(\frac{V^2 y^2 b^2}{gb^2} \right)^{1/3} = \left(\frac{(14 \text{ ft/s})^2 (0.8 \text{ ft})^2}{(32.2 \text{ ft/s}^2)} \right)^{1/3} = 1.57 \text{ ft}$

(c) The flow is **supercritical** since $Fr > 1$.

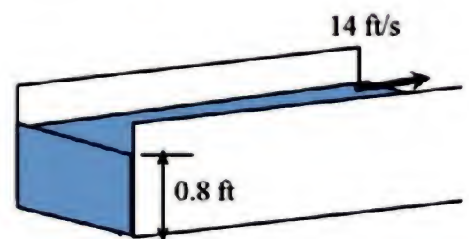
For the case of $y = 0.2 \text{ ft}$:

Replacing 0.8 ft in above calculations by 0.2 ft gives

$$Fr = \frac{V}{\sqrt{gy}} = \frac{14 \text{ ft/s}}{\sqrt{(32.2 \text{ ft/s}^2)(0.2 \text{ ft})}} = 5.52$$

$$y_c = \left(\frac{\dot{V}^2}{gb^2} \right)^{1/3} = \left(\frac{V^2 y^2 b^2}{gb^2} \right)^{1/3} = \left(\frac{(14 \text{ ft/s})^2 (0.2 \text{ ft})^2}{(32.2 \text{ ft/s}^2)} \right)^{1/3} = 0.625 \text{ ft}$$

The flow is supercritical in this case also since $Fr > 1$.



Example 2.3: Water flows at a depth of 0.8 ft with an average velocity of 10 ft/s in a wide rectangular channel. Determine

(a) The Froude number, (b) The critical depth, (c) whether the flow is subcritical or supercritical, and (d) what would your response be if the flow depth were 0.2 ft?

Ex(2.3):

a) $Fr = ?$
b) $y_c = ?$
c) sub or super?
d) $y = 0.2$ ft

if $y_2 = ?$

$V = 10$ ft/s
 $y = 0.8$ ft

a) $Fr = \frac{V}{\sqrt{gy}} = \frac{10}{\sqrt{32.2 \times 0.8}} = 1.97 > 1$

$\therefore y = y_1 = 0.8$ ft

b) $y_c = \left(\frac{q^2}{g} \right)^{1/3}$ | $q = yV$
 $q = 0.8 \times 10$
 $q = 8$ ft²/s

$y_c = \left(\frac{8^2}{32.2} \right)^{1/3}$

$y_c = 1.26$ ft

c) $Fr > 1 \therefore$ supercritical flow

d) $y_1 = 0.2$ ft $Fr = \frac{10}{\sqrt{32.2 \times 0.2}} = 3.94$

$q = yV = 0.2 \times 10 = 2$ ft²/s

$y_c = \left(\frac{2^2}{32.2} \right)^{1/3} = 0.5$ ft

if $y_2 = ?$

$E_1 = 0.8 + \frac{8^2}{2(32.2)(0.8)^2} = 2.35$ ft

$2.35 = y_2 + \frac{8^2}{2(32.2)y_2^2}$

$\therefore y_2 = 2.13$ ft

Example 2.4: water flows through a 4 m wide rectangular channel with an average velocity of 7 m/sec. if the flow is critical, determine the flow rate.

EX (2.4):

Critical flow:

$V = 7 \text{ m/s}$

$b = 4 \text{ m}$

$Q = ?$

$\therefore y = y_c$

$Fr = 1$

$q = \frac{Q}{b} \Rightarrow Q = qb \dots \textcircled{1}$

$q = y_c V \dots \textcircled{2}$

$q = 5 \times 7 = 35 \text{ m}^2/\text{s}$

$\therefore Q = 35 \times 4 = 140 \text{ m}^3/\text{s}$

$Fr = \frac{V}{\sqrt{gy_c}} = 1 = \frac{V}{\sqrt{gy_c}}$

$V^2 = (\sqrt{gy_c})^2$

$V^2 = g y_c \Rightarrow y_c = \frac{V^2}{g}$

$y_c = \frac{7^2}{9.81} = 4.99 \approx \underline{5 \text{ m}}$

Example 2.5: water flows at a depth of 0.4 m with an average velocity of 6 m/sec in a rectangular channel. Determine (a) the critical depth, (b) the alternate depth, and (c) the minimum specific energy.

EX (2.5):

$V = 6 \text{ m/s}$

$y = 0.4 \text{ m}$

a) $y_c = ?$

b) $y_2 \text{ (alternate)} = ?$

c) $E_{\min} = ?$

$Fr = \frac{6}{\sqrt{9.81 \times 0.4}} = 3.03 > 1$

$y = y_1 = 0.4 \text{ m}$

a) $y_c = \left(\frac{q^2}{g} \right)^{1/3}$

$q = y V$

$q = 0.4 \times 6 = 2.4 \text{ m}^2/\text{s}$

$y_c = \left(\frac{2.4^2}{9.81} \right)^{1/3} = \underline{0.84 \text{ m}}$

b) $E_1 = y_1 + \frac{q^2}{2gy_1^3}$

$E_1 = 0.4 + \frac{2.4^2}{2(9.81)(0.4)^3} = 2.23 \text{ m} = E_2$

$2.23 = y_2 + \frac{2.4^2}{2(9.81)y_2^3}$

$y_2 = 2.17 \text{ m}$

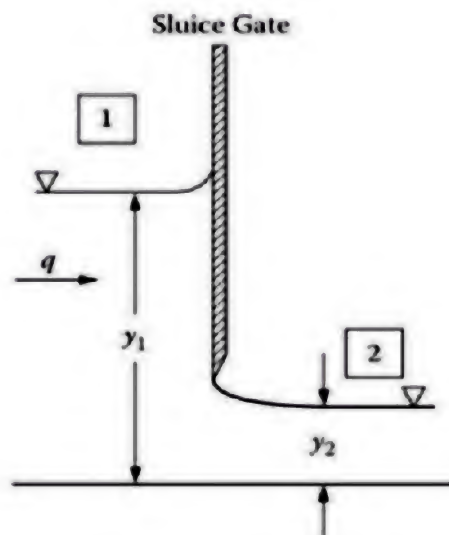
c) $E_{\min} = ?$

$E_{\min} = \frac{3}{2} y_c$

$E_{\min} = \frac{3}{2} \times 0.84 = 1.26 \text{ m}$

Example 2.6: The system shown in figure below has a specific discharge of 3 m²/s. The depth, y_1 , upstream of the sluice gate is 2 meters. Determine:

- The upstream Froude number, Fr_1 ,
- The specific energy upstream of the sluice gate, E_1 .
- The downstream depth, y_2 .
- The downstream specific energy, E_2 .
- The downstream Froude number, Fr_2 .



Solution:

- a. Noting that $q = v \cdot y_1$, the upstream Froude number is:

$$F_{r,1} = \frac{v}{\sqrt{gy_1}} = \frac{q}{\sqrt{gy_1^3}} = \frac{3.0}{\sqrt{(9.81)(2.0)^3}} = 0.34$$

Notice that $F_{r,1} < 1$, so this confirms our expectation that the Froude number upstream of the gate is subcritical.

- b. Specific energy upstream of the sluice gate is:

$$E_1 = \frac{q^2}{2gy_1^2} + y_1 = \frac{(3.0)^2}{2(9.81)(2.0)^2} + 2.0 = 2.11 \text{ m}$$

Notice that this result is visually consistent with the right side of Figure 2.6, which indicates an energy slightly greater than 2 m.

- c. The downstream depth is (using equation 2.22 and the result from Example 2.1a):

$$y_2 = \frac{2(2.0)}{-1 + \sqrt{1 + \frac{8}{(0.34)^2}}} = 0.54 \text{ m}$$

- d. The downstream specific energy is:

$$E_2 = \frac{q^2}{2gy_2^2} + y_2 = \frac{(3.0)^2}{2(9.81)(0.54)^2} + 0.54 = 2.11 \text{ m}$$

Notice the $E_1 = E_2$ as we would expect since y_1 and y_2 are alternate depths.

- e. The downstream Froude number is:

$$F_{r,2} = \frac{q}{\sqrt{gy_2^3}} = \frac{3.0}{\sqrt{(9.81)(0.54)^3}} = 2.41$$

$F_{r,2} > 1$, confirming our expectation that the Froude number downstream of the gate is supercritical.

Example 2.7: The system shown in figure (2-5) has a specific discharge of 3 m²/s. The depth, y_1 , upstream of the step is 2 meters. The upward step height is 0.2 meters.

- Determine the downstream specific energy, E_2 .
- Determine the downstream depth, y_2
- Determine the absolute change in the water surface from location 1 to location 2.
- Determine the downstream Froude number, Fr_2
- Interpret the results in the context of an E - y diagram.

Solution:

- The flow conditions upstream of the step are the same as in Example 2.1, where it was previously determined that $E_1 = 2.11$ meters. Therefore,

$$E_2 = E_1 - \Delta z = 2.11 - 0.2 = 1.91 \text{ m}$$

Thus, a flow encountering an upward step can be thought of as paying an “energy tax” equal to the step height in order to continue flowing downstream.

- The downstream depth, y_2 , is determined by trial and error using the specific energy equation:

$$E_2 = \frac{q^2}{2gy_2^2} + y_2 = \frac{(3.0)^2}{2(9.81)y_2^2} + y_2 = 1.91 \text{ m}$$

There are two positive solutions for y_2 : 0.59 m and 1.76 m. These two depths are an alternate depth pair that both produce a specific energy of 1.91 m. As the conditions upstream of the step are subcritical, the solution will proceed with the subcritical value for $y_2 = 1.76$ m, but there will be a need for further discussion of the rationale for this choice outside of this example problem.

- c. The absolute change in water depth, Δw_s , between locations 1 and 2 is determined from a quick examination of the solved system:

$$\Delta w_s = y_2 + \Delta z - y_1 = 1.76 + 0.2 - 2.0 = -0.04 \text{ m}$$

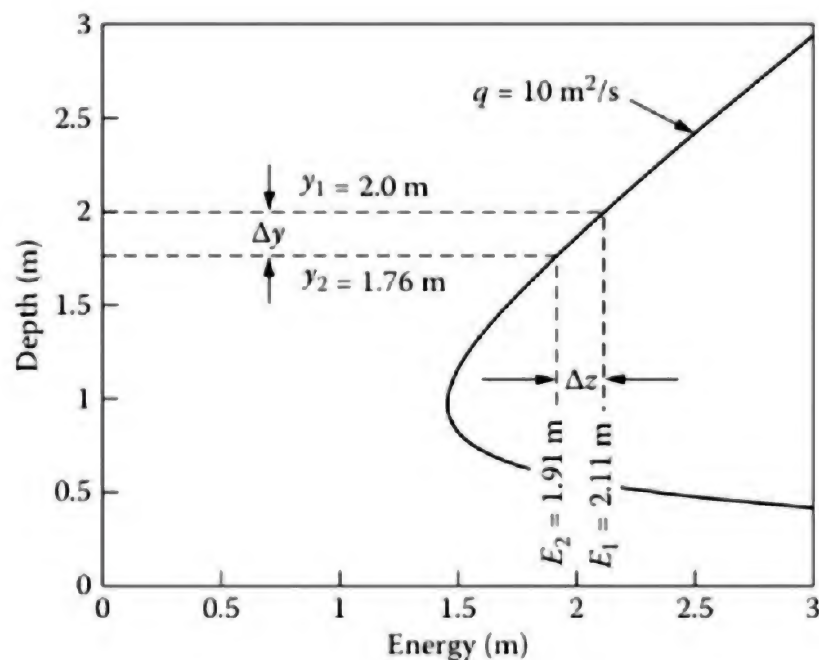
Thus, the additional presence of the step serves to actually *lower* the water surface slightly, by 0.04 m.

- d. The downstream Froude number, $F_{r,2}$ is:

$$F_{r,2} = \frac{q}{\sqrt{gy_2^3}} = \frac{3.0}{\sqrt{(9.81)(1.76)^3}} = 0.41$$

Comparing the upstream (calculated in Example 2.1 as 0.34) and downstream Froude numbers shows that the Froude number remains subcritical but increases in the downstream direction. This indicates that the effect of the upward step is to drive the flow closer to critical conditions.

- e. Figure 2.8 shows the E - y diagram that corresponds to Example 2.2. The “energy tax” alluded to in Example 2.2a corresponds physically to a horizontal shift from E_1 to E_2 equal to the height of the step.



Example 2.8: Consider a system with a specific discharge of $3 \text{ m}^2/\text{s}$. The depth, y_1 , upstream of the step is 2 meters. The downward step height is 0.2 meters.

- Determine the downstream specific energy, E_2 .
- Determine the downstream depth, y_2
- Determine the absolute change in the water surface from location 1 to location 2.
- Determine the downstream Froude number, Fr_2
- Contrast the results of this example with the results from example 2.7.

Solution:

- The flow conditions upstream of the step are the same as in Example 2.1, where it was previously determined that $E_1 = 2.11$ meters. Therefore,

$$E_2 = E_1 + \Delta z = 2.11 + 0.2 = 2.31 \text{ m}$$

Thus, a flow encountering a downward step can be thought of as receiving an “energy gift” equal to the step height.

- The downstream depth, y_2 , is determined by trial and error using the specific energy equation:

$$E_2 = \frac{q^2}{2gy_2^2} + y_2 = \frac{(3.0)^2}{2(9.81)y_2^2} + y_2 = 2.31 \text{ m}$$

There are two positive solutions for y_2 : 0.50 m and 2.22 m. These two depths are an alternate depth pair that both produce a specific energy of 2.31 m. As in Example 2.2, the solution will proceed with the subcritical value for $y_2 = 2.22 \text{ m}$.

- c. The absolute change in water depth, Δw_s , between locations 1 and 2 is:

$$\Delta w_s = y_2 - \Delta z - y_1 = 2.22 - 0.2 - 2.0 = 0.02 \text{ m}$$

TABLE 2.2 Summary and Contrast in Findings from Examples 2.2 and 2.3

Quantity	Example 2.2	Example 2.3
Step height, Δz	+0.2 m	-0.2 m
$E_1 - E_2$	+0.2 m	-0.2 m
y_1 relative to y_2	$y_1 > y_2$	$y_1 < y_2$
Δw_s	-0.04 m	+0.02 m
$F_{r,1}$ relative to $F_{r,2}$	$F_{r,1} < F_{r,2}$	$F_{r,1} > F_{r,2}$

Thus, the downward step results in a *higher* water surface by 0.02 m.

- d. The downstream Froude number, $F_{r,2}$ is:

$$F_{r,2} = \frac{q}{\sqrt{gy_2^3}} = \frac{3.0}{\sqrt{(9.81)(2.22)^3}} = 0.29$$

Comparing the upstream (calculated in Example 2.1 as 0.34) and downstream Froude numbers shows that the Froude number becomes more subcritical. That is, the downward step serves to drive the flow away from critical conditions.

- e. Contrasting Example 2.3 with Example 2.2 shows, not surprisingly, that the effect of a subcritical flow encountering a downward step is generally opposite to that of the same flow encountering an upward step. Results are summarized in Table 2.2.

Example 2.9: consider a system with a discharge of $9 \text{ m}^3/\text{sec}$. the channel is rectangular. The width at location 1 is $w_1=4.5 \text{ m}$. A constriction is encountered at location 2 downstream such that the width $w_2 = 3 \text{ m}$. The depth of flow y_2 at downstream location 2 is 2 meters.

- Determine the specific discharge at locations 1 and 2.
- Determine the downstream specific energy, E_2 .
- Determine the downstream Froude number, Fr_2 .
- Determine the upstream specific energy, E_1 .
- Determine the upstream depth, y_1 .
- Determine the upstream Froude number, Fr_1 .
- Determine the absolute change in the water surface from location 1 to location 2.
- Sketch the transition of the system from location 1 through the flow constriction to location 2 on an E-y diagram.

Solution:

- The specific discharge are:

$$q_1 = \frac{Q}{w_1} = \frac{9}{4.5} = 2.0 \frac{\text{m}^2}{\text{sec}}, \quad q_2 = \frac{Q}{w_2} = \frac{9}{3} = 3.0 \frac{\text{m}^2}{\text{sec}}$$

- The specific energy, E_2 , is the same as the specific energy, E_1 . Where $E_1 = 2.11 \text{ m}$.
- The Froude number, Fr_2 , is the same as the Froude number Fr_1 . $Fr_2 = 0.34$.
- The constriction is energy conserving, thus, $E_1 = E_2 = 2.11 \text{ m}$
- The flow depth upstream, y_1 , must satisfy the specific energy equation for the energy calculated above and the specific discharge calculated:

$$E_1 = \frac{q_1^2}{2g y_1^2} + y_1 = \frac{2^2}{2(9.81)y_1^2}$$

By trial and error, two positive solutions for y_1 exist: 0.34 m and 2.06 m. these two depths are an alternate depth pair that both produce a specific energy of 2.11 m. By the same reasoning earlier, the solution will proceed with the subcritical value for $y_2 = 2.06 \text{ m}$.

- The upstream Froude number, Fr_1 is:

$$Fr_1 = \frac{q_1}{\sqrt{g y_1^3}} = \frac{2}{\sqrt{9.81 \cdot 2.03^3}} = 0.22$$

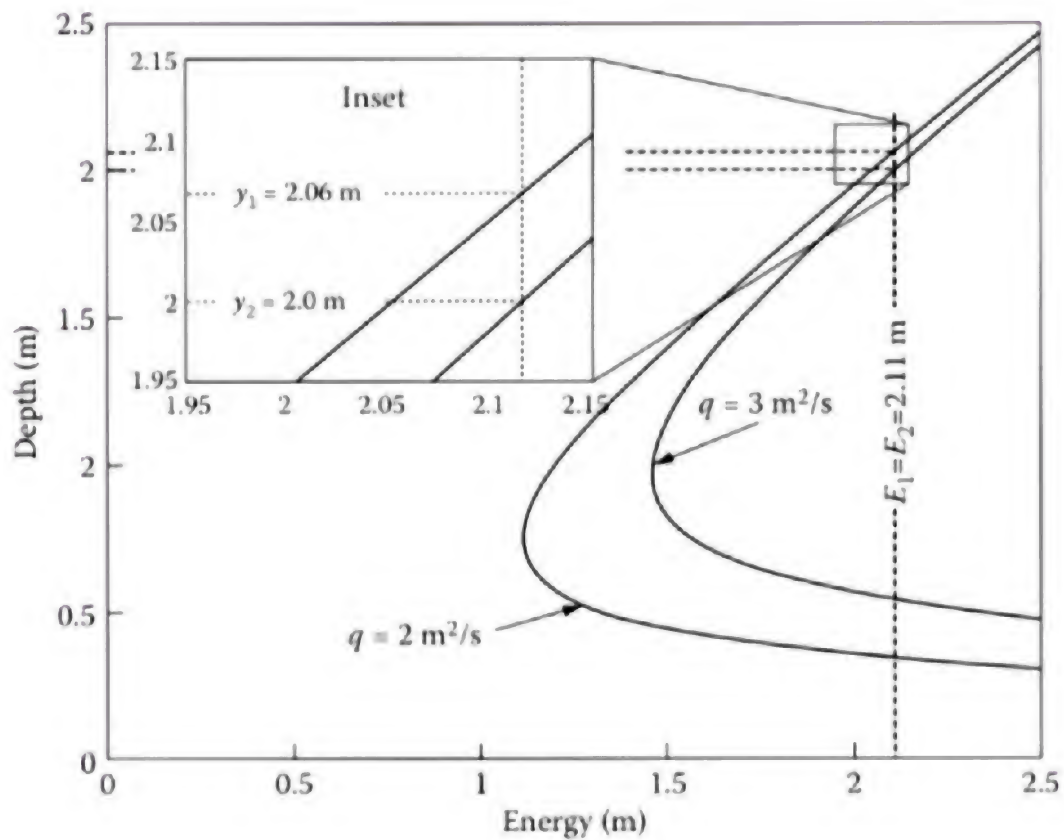
Thus the constriction drives the flow toward critical conditions.

g. The absolute change in water depth, Δw_s , between locations 1 and 2 is:

$$\Delta w_s = y_2 - y_1 = 2 - 2.06 = -0.06 \text{ m}$$

Thus the flow depth has decreased 0.06 m within the constriction.

h. Figure below shows an E-y diagram with q –curves corresponding to $q_1 = 2 \text{ m}^2/\text{sec}$ curve at an energy of 2.11 m and depth of 2.06 m down to the $q_2 = 3 \text{ m}^2/\text{sec}$ curve at the same energy 2.11 m and a depth of 2 m.



Example 2.10: consider a system with a discharge of $9 \text{ m}^3/\text{sec}$. the channel is rectangular. The width at location 1 is $w_1 = 4.5 \text{ m}$ with flow depth $y_1 = 2.06 \text{ m}$. A constriction is encountered at location 2 downstream such that the width $w_2 = 1.5 \text{ m}$.

- Determine the energy initially held by the flow at location 1.
- Determine the minimum energy required to flow through the constriction at location 2.
- Assuming the flow as initially specified instantaneously encounters the constriction at location 2, determine the initial discharge that passes through the constriction.
- Assuming the flow as initially specified instantaneously encounters the constriction at location 2 determine the initial rate of water storage immediately upstream of the constriction.
- Determine the depths and specific energy values at locations 1 and 2 when the system returns to steady state.

Solution:

$$\text{a- } E_1 = y_1 + \frac{q_1^2}{2g y_1^3} = 2.11 \text{ m}$$

b- At location 2, the new width of the channel is 1.5 m . Thus,

$q_{2,\text{new}} = 9 / 1.5 = 6 \text{ m}^2/\text{sec}$. knowing that the minimum energy corresponds to critical conditions,

$$E_{\min} = E_c = \frac{3}{2} y_c = \frac{3}{2} \left(\frac{q_{2,\text{new}}^2}{g} \right)^{\frac{1}{3}} = 2.31 \text{ m}$$

So the specific energy at location 1 is approximately 0.2 m smaller than the required energy to pass location 2. This means location 2 is a **choke** given the specified initial conditions.

- c- The initial discharge that passes location 2 is the maximum discharge that can be supported by a specific energy of 2.11 m . Its known for a given E , discharge is maximized at critical conditions. Thus, $E_c = 2.11 \text{ m}$. our goal is to determine what q_c and Q_c correspond to this critical energy.

$$y_{c,\text{init}} = \frac{2}{3}(2.11) = 1.41 \text{ m}$$

$$q_{c,\text{init}} = \sqrt{g y_{c,\text{init}}^3} = \sqrt{(9.81)(1.41)^3} = 5.19 \frac{\text{m}^2}{\text{s}}$$

Since $Q = q \cdot w$,

$$Q_{c,\text{init}} = (5.19)(1.5) = 7.79 \frac{\text{m}^3}{\text{s}}$$

- d. The initial rate of storage upstream of the constriction is the difference between the steady-state flow rate ($Q_{ss} = 9 \text{ m}^3/\text{s}$) and the initial discharge ($Q_{c,init} = 7.79 \text{ m}^3/\text{s}$) calculated immediately above:

$$\frac{\Delta S}{dt} = Q_{ss} - Q_{c,init} = 9.0 - 7.79 = 1.21 \frac{\text{m}^3}{\text{s}}$$

Note that this is the initial rate of storage at the very moment the choke is first encountered. Since this discharge is being stored, the depth will be backing up upstream of the constriction, resulting in increased specific energy upstream of the constriction. So the transient will be characterized by a diminishing storage rate as the specific energy deficit decreases from 0.20 m to 0 when the system returns to steady state.

- e. When the system returns to steady state, the discharge through the constriction in location 2 will be $9 \text{ m}^3/\text{s}$. The system will only accrue the minimum energy to pass this discharge, so location 2 will still be at critical conditions. At location 2, this will mean that $q = 9/1.5 = 6 \text{ m}^2/\text{s}$. The critical depth, $y_{c,ss}$ and specific energy, $E_{c,ss}$, associated with this specific discharge are:

$$y_{2,ss} = y_{c,ss} = \left(\frac{q_2^2}{g} \right)^{1/3} = \left(\frac{(6.0)^2}{9.81} \right)^{1/3} = 1.54 \text{ m}$$

$$E_{2,ss} = E_{c,ss} = \frac{3}{2} y_{c,ss} = \frac{3}{2} (1.54) = 2.31 \text{ m}$$

To determine the depth and specific energy upstream at location 1, we note that specific energy is conserved between locations 1 and 2 and recall that the width at the upstream location is 4.5 m, so $q_1 = 2 \text{ m}^2/\text{s}$ and:

$$E_{1,ss} = E_{2,ss} = 2.31 \text{ m}$$

$$E_1 = \frac{q_1^2}{2gy_{1,ss}^2} + y_{1,ss} = \frac{(2.0)^2}{2(9.81)y_{1,ss}^2} + y_{1,ss} = 2.31$$

Solving this latter equation for $y_{1,ss}$ and taking the subcritical root, we get $y_{1,ss} = 2.27 \text{ m}$. Compare this final, steady-state depth with the initial depth before the choke was resolved of $y_{1,init} = 2.06 \text{ m}$. Thus, the increase in upstream flow depth is about 0.21 m. This increase in depth is by virtue of the choke and the subsequent backup of discharge during the transient recovery and return to a steady-state discharge of $9 \text{ m}^3/\text{s}$.

Example 2.11: Consider a trapezoidal section with side slope parameter $m = 2$, and bottom width $b = 1.5$ m. The discharge is $Q = 9$ m³/s. Let the depth be $y_1 = 2$ m.

- a- Determine the cross sectional area
- b- Determine the top width, T or B
- c- Determine the specific energy, E_1
- d- Determine the alternate depth, y_2

Solutions:

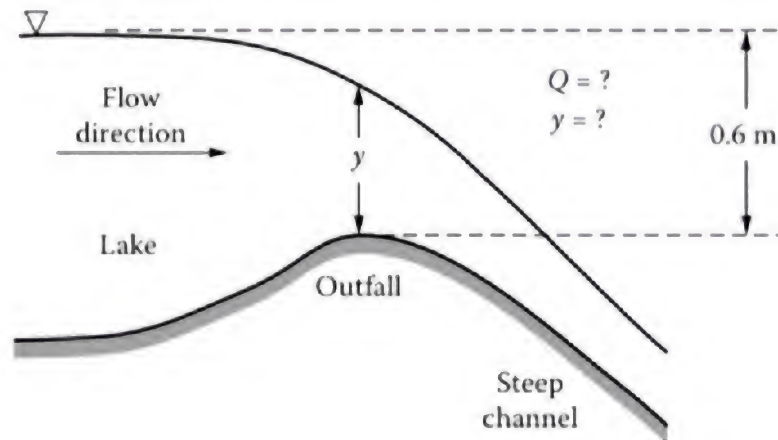
a- $A_1 = by + my^2 = (1.5) \cdot (2.0) + 2 \cdot (2.0)^2 = 11.0 \text{ m}^2$

b- $B_1 = b + 2my = 1.5 + (2) \cdot (2) \cdot (2.0) = 9.5 \text{ m}$

c- $E_1 = \frac{Q^2}{2gA_1^2} + y = \frac{9^2}{2 \cdot (9.81) \cdot (11.0)^2} + 2.0 = 2.03 \text{ m}$

- d- The alternate depth, y_2 , can be determined by iteration. Values of y_2 are assumed and entered into equation 2.4.
 $y_2 = 0.60 \text{ m}.$

Example 2.12: a trapezoidal channel (bottom width 1.5 m, side slopes $m=2$) forms the outlet structure of a lake that discharges to a steep channel as shown in figure below. The lake level far from the outlet is 0.6 relative to the invert of the outlet structure. What is the discharge from the lake?



Solution: the flow can be assumed to be critical at the head of the steep channel. From the problem statement the critical energy associated with the flow at the outlet is 0.6 m.

Now using the figure 2.11, the vertical axis components are given in the question:

$$\frac{mE_c}{b} = \frac{(2) \cdot (0.6)}{1.5} = 0.8$$

Entering the vertical axis at 0.8, we move to the trapezoidal section curve and down to the horizontal axis where we read approximately 0.6. Thus,

$$0.6 = \frac{Qm^{3/2}}{b^2 \sqrt{gb}}$$

Rearranging and solving for Q ,

$$Q = \frac{(0.6) \cdot b^2 \sqrt{gb}}{m^{3/2}} = \frac{(0.6) \cdot (1.5) \cdot \sqrt{(9.81) \cdot (1.5)}}{2^{3/2}} = 1.8 \frac{\text{m}^3}{\text{s}}$$

Additionally the discharge can be determined using the equation 2.5:

$$Q = \sqrt{2gA^2(E_c - y)}$$

Expanding for A using the equation for area of a trapezoid,

$$Q = \sqrt{2g \cdot (by + my^2)^2 (E_c - y)}$$

Gives Q as a function of depth, y and its profound that discharge is maximized at critical conditions. Assuming values of y , we can tabulate the above equation for $b = 1.5$ m, and $E_c = 0.6$ m to determine Q . the value of y that maximizes Q corresponds to critical depth and is the solution to this problem.

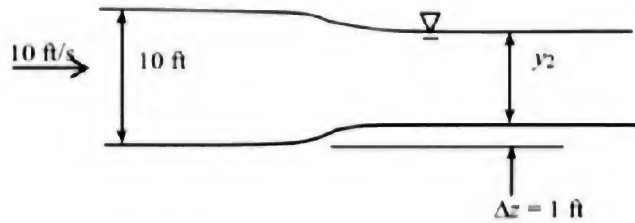
Table 2.3 Variation in Area, Top Width, Froude Number, and Discharge with Depth

Depth, y (m)	Area, A (m ²)	Top Width, B (m)	Froude Number, F_r	Discharge, Q (m ³ /s)
0.41	0.951	3.14	1.120	1.837
0.42	0.983	3.18	1.079	1.847
0.43	1.015	3.22	1.039	1.853
0.44	1.047	3.26	0.998 (essentially 1)	1.855 (maximum)
0.45	1.080	3.30	0.957	1.853
0.46	1.113	3.34	0.917	1.845
0.47	1.147	3.38	0.875	1.832

Thus, the more precise method that yields an estimate of $Q = 1.855$ m³/s is consistent with $Q = 1.8$ m³/s derived from Figure 2.11.

Example 2.13:

Water is flowing at a depth of 10 ft with a velocity of 10 ft/s in a channel of rectangular section. Find the depth and change in water surface elevation caused by a smooth upward step in the channel bottom of 1 ft. What is the maximum allowable step size so that choking is prevented? (Use a head loss coefficient = 0.)



$$q = V_1 y_1 = (10)(10) = 100 \text{ ft}^2/\text{s}$$

$$y_c = \left(\frac{q^2}{g} \right)^{1/3} = \left(\frac{100^2}{32.2} \right)^{1/3} = 6.77 \text{ ft} \Rightarrow \text{subcritical approach flow}$$

$$E_c = \frac{3}{2} y_c = (1.5)(6.77) = 10.16 \text{ ft}$$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 10 + \frac{10^2}{64.4} = 11.55 \text{ ft}$$

Now because $E_1 - \Delta z > E_c$, there is no choking. So write the energy equation from 1 to 2 and find a subcritical solution for y_2 :

$$E_1 = y_2 + \frac{q^2}{2gy_2^3} + \Delta z$$

$$11.55 = y_2 + \frac{100^2}{64.4y_2^3} + 1.0$$

$$y_2 + \frac{155.3}{y_2^3} = 10.55$$

which can be solved by trial and error or an equation solver to give $y_2 = 8.29$ ft. The water surface elevation drops by $10 - (8.29 + 1) = 0.71$ ft.

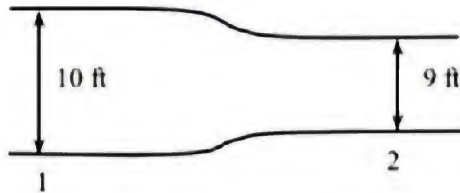
For the limiting choking case, set the specific energy at section 2 equal to E_c :

$$E_1 = 11.55 = E_c + \Delta z_c = 10.16 + \Delta z_c$$

$$\text{so } \Delta z_c = 11.55 - 10.16 = 1.39 \text{ ft.}$$

Example 2.14:

The upstream conditions are the same as in Exercise 2.1, but there is a smooth contraction in width from 10 ft to 9 ft and a horizontal bottom. Find the depth of flow and change in water surface elevation in the contracted section. What is the greatest allowable contraction in width so that choking is prevented? (Head loss coefficient = 0.)



From Exercise 2.1, $q_1 = 100$ cfs/ft; $y_{c1} = 6.77$ ft; and $E_1 = 11.55$ ft. Then from continuity, we have $q_2 = (10/9) q_1 = (10/9)(100) = 111.1$ cfs/ft and $y_{c2} = (111.1^2/32.2)^{1/3} = 7.26$ ft. Writing the energy equation from 1 to 2:

$$E_1 = y_2 + \frac{q_2^2}{2gy_2^2}$$
$$11.55 = y_2 + \frac{(111.1)^2}{64.4y_2^2} = y_2 + \frac{191.7}{y_2^2}$$

from which the subcritical solution is $y_2 = 9.36$ ft with a water surface elevation drop of $(10 - 9.36) = 0.64$ ft.

For the limiting choking case, $E_1 = E_{c2} = 1.5 y_{c2}$, so that

$$11.55 = 1.5 \left(\frac{q_2^2}{g} \right)^{1/3}$$
$$q_2 = [(2/3)(11.55)]^{3/2} (32.2)^{1/2} = 121.2 \text{ cfs/ft}$$

But $b_2 = Q/q_2 = 1000/121.2 = 8.25$ ft.

Example 2.15:

Determine the downstream depth in the transition and the change in water surface elevation if the channel bottom rises 0.15 m and the upstream conditions are a velocity of 4.5 m/s and a depth of 0.6 m.

$$q = (4.5)(0.6) = 2.7 \text{ m}^2/\text{s}$$

$$y_c = \left(\frac{q^2}{g} \right)^{1/3} = \left(\frac{2.7^2}{9.81} \right)^{1/3} = 0.906 \text{ m}$$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 0.6 + \frac{4.5^2}{19.62} = 1.63 \text{ m}$$

The approach flow is supercritical, and the minimum specific energy, $E_c = 1.5y_c = (1.5)(0.906) = 1.36 \text{ m}$. Because $E_1 - \Delta z = 1.63 - 0.15 = 1.48 \text{ m} > E_c$, choking is not expected. Solve the energy equation for y_2 in the supercritical flow regime:

$$E_1 = 1.63 = y_2 + \frac{q^2}{2gy_2^2} + \Delta z = y_2 + \frac{2.7^2}{19.62y_2^2} + 0.15$$

$$y_2 + \frac{0.3716}{y_2^2} = 1.48$$

and the solution is $y_2 = 0.683 \text{ m}$. The water surface elevation rises by $0.683 + 0.15 - 0.60 = 0.233 \text{ m}$. The limiting choking case is given by $\Delta z = E_1 - E_c = 1.63 - 1.36 = 0.27 \text{ m}$.

As in Figure 2.9 for a width contraction with a supercritical approach flow, there is a second mode of choking in this example with a hydraulic jump upstream of the transition and critical depth in the transition. As discussed in Chapter 3, the sequent depth for an upstream hydraulic jump can be calculated to be 1.30 m corresponding to a specific energy after the jump of $E_1 = 1.52 \text{ m}$; however, in this event, $E_1 - \Delta z = 1.37 \text{ m}$, which is still greater than E_c , so no choking occurs by this mode either.

Chapter Three

Example

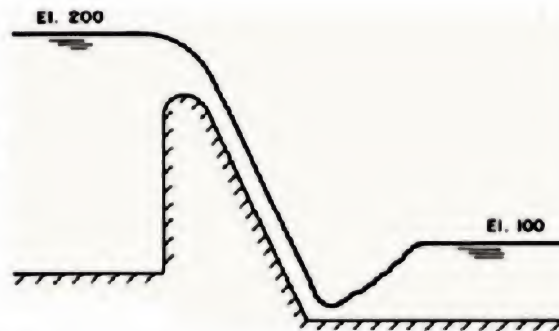
The reservoir level upstream of a 30-m wide spillway for a flow of $800 \text{ m}^3/\text{s}$ is at El. 200 m. The downstream river level for this flow is at El. 100 m. Determine the invert level of a stilling basin having the same width as the spillway so that a hydraulic jump is formed in the basin. Assume the losses in the spillway are negligible.

$$Q = 800 \text{ m}^3/\text{s}$$

$$B = 30 \text{ m}$$

Upstream water level = El 200 m

Downstream water level = El 100



Determine:

Stilling basin invert elevation to form the jump ?

Solution:

Let z be the invert elevation of the stilling basin. Then, $y_2 = 100 - z$. Since the losses on the spillway face are negligible and assuming y_1 to be small,

$$V_1 = \sqrt{2g(200 - z)}$$

Now, $Q = BV_1y_1$. Hence

$$\begin{aligned} y_1 &= \frac{800}{30 \times \sqrt{19.62(200 - z)}} \\ &= \frac{6.02}{\sqrt{200 - z}} \end{aligned}$$

Substituting expressions for y_1 and V_1

$$\begin{aligned} F_{r1}^2 &= \frac{V_1^2}{gy_1} \\ &= \frac{19.62(200 - z)}{9.81 \times 6.02 / \sqrt{200 - z}} \\ &= 0.332(200 - z)^{1.5} \end{aligned}$$

$$\frac{100 - z}{6.02\sqrt{200 - z}} = \frac{1}{2} \left(-1 + \sqrt{1 + 8 \times 0.332(200 - z)^{1.5}} \right)$$

$$(100 - z)\sqrt{200 - z} = -3.01 + 3.01\sqrt{1 + 2.656(200 - z)^{1.5}}$$

$$z = 84.18 \text{ m}$$

Thus, the stilling basin invert should be at **El. 84.18** to form the jump.

Example

A hydraulic jump is formed in a 5-m wide outlet at a short distance downstream of a control gate (Fig. 2-13). If the flow depths just downstream of the gate is 2 m and the outlet discharge is 150 m³/s, determine

i. Flow depth downstream of the jump;

ii. Thrust on the gate; and

iii. Head losses in the jump.

Assume there are no losses in the flow through the gate.

Handwritten solution for the hydraulic jump problem:

Given:
 $B = 5 \text{ m}$
 $y_2 = 2 \text{ m}$
 $Q = 150 \text{ m}^3/\text{s}$

Find:
i) $y_3 = ?$
ii) $F_g = ?$
iii) ΔE

Diagram: A schematic of a hydraulic jump. A vertical gate is shown. Upstream of the gate, the flow depth is y_1 . Just downstream of the gate, the flow depth is $y_2 = 2 \text{ m}$. Further downstream, a hydraulic jump occurs, and the flow depth becomes $y_3 = ?$. The flow is from left to right.

Energy and Force Equations:
 $E_1 = E_2 \neq E_3$
 $F_{s1} \neq F_{s2} = F_{s3}$

Calculation for y_3 :
$$\frac{y_3}{y_2} = \frac{1}{2} (-1 + \sqrt{1 + 8 Fr_{1,2}^2})$$

$$Fr_{1,2} = \frac{V_2}{\sqrt{g y_2}} = \frac{Q}{A \sqrt{g y_2}} = \frac{150}{2 \times 5 \sqrt{9.81 \times 2}} = 3.386$$

$$y_3 = \frac{1}{2} (2) (-1 + \sqrt{1 + 8 (3.386)^2})$$

$$\boxed{y_3 = 8.63 \text{ m}}$$

$$F_g = \gamma (F_{s,1} - F_{s,2})$$

$$F_{s,2} = \frac{Q^2}{g A_2} + \bar{z}_2 A_2 = \frac{150^2}{9.81(2 \times 5)} + \frac{1}{2}(2)(2 \times 5)$$

$$F_{s,2} = 239.36$$

$$F_{s,1} = \frac{Q^2}{g A_1} + \bar{z}_1 A_1 = \frac{Q^2}{g(y_1 B)} + \frac{1}{2} y_1 (y_1 B) \dots \text{--- (1)}$$

$$E_2 = y_2 + \frac{V^2}{2g} = y_2 + \frac{Q^2}{(y_2 B)^2 2g} = 2 + \frac{150^2}{2(9.81)(2 \times 5)^2}$$

$$E_2 = 13.47 \text{ m} = E_1$$

$$13.47 = y_1 + \frac{Q^2}{2g(y_1 B)^2}$$

$$13.47 = y_1 + \frac{150^2}{2(9.81)5^2 y_1^2}$$

$$y_1 = 13.21 \text{ m}$$

$$\therefore F_{s,1} = \frac{150^2}{9.81(13.21 \times 5)} + \frac{1}{2}(13.21)(13.21 \times 5)$$

$$F_{s,1} = 470.985$$

$$\therefore F_g = 9.81(470.985 - 239.36)$$

$$F_g = 2272.24125 \text{ kN}$$

$$\text{iii) } \Delta Z = E_{1,2} - E_3 \dots \text{--- (1)}$$

$$E_3 = y_3 + \frac{Q^2}{2g A^2}$$

$$E_3 = 8.63 + \frac{150^2}{2(9.81)(8.63 \times 5)^2}$$

$$E_3 = 9.25 \text{ m}$$

$$\Delta Z = 13.47 - 9.25$$

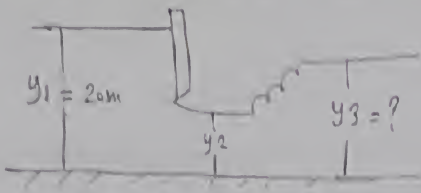
$$\Delta Z = 4.22 \text{ m}$$

2.8. A hydraulic jump is formed in a 4-m wide outlet just downstream of the control gate, which is located at the upstream end of the outlet. The flow depth upstream of the gate is 20 m. If the outlet discharge is $100 \text{ m}^3/\text{s}$, determine

- Flow depth downstream of the jump;
- Thrust on the gate; and
- Energy losses in the jump.

Assume there are no losses in the flow through the gate.

$B = 4 \text{ m}$
 $y_1 = 20 \text{ m}$
 $Q = 100 \text{ m}^3/\text{s}$
 $y_3 = ?$
 $F_g = ?$
 $\Delta E = ?$



$E_1 = E_2 = E_3$
 $F_{s1} = F_{s2} = F_{s3}$

$\frac{y_3}{y_2} = \frac{1}{2} (-1 + \sqrt{1 + 8 Fr_2^2}) \quad \dots (1)$
 $E_1 = 20 + \frac{100^2}{(20 \cdot 4)^2 2(9.81)} = 20.08 \text{ m} \quad 20.08 = y_2 + \frac{100^2}{2(9.81) y_2^2 (4)^2}$
 $y_2 = 1.3 \text{ m} \Rightarrow Fr_{2,2} = \frac{V_2}{\sqrt{g y_2}} = \frac{Q}{A \sqrt{g y_2}} = \frac{100}{1.3 \times 4 \sqrt{9.81 \times 1.3}}$
 $Fr_{2,2} = 5.385 > 1 \quad \therefore \checkmark$
 $y_3 = \frac{1}{2} (-1 + \sqrt{1 + 8 (5.385)^2}) \times 1.3 = 9.27 \text{ m}$
 $F_g = Y (F_{s1} - F_{s(2,3)}) \quad \dots (1) \quad F_{s,2} = \frac{Q^2}{g A_2} + \bar{z}_2 A_2 = \frac{100^2}{9.81 (1.3 \times 4)} + \frac{1}{2} (1.3) (1.3 \times 4)$
 $F_{s,2} = 199.41 \quad F_{s1} = \frac{Q^2}{g A_1} + \bar{z} A_1 = \frac{100^2}{9.81 (20 \times 4)} + \frac{1}{2} (20) (20 \times 4)$
 $F_{s1} = 812.74$
 $\therefore F_g = 9.81 (812.74 - 199.41) = 6016.7673 \text{ kN}$
 $\Delta E = E_{1,2} - E_3 \quad \dots (1) \quad E_3 = 9.27 + \frac{100^2}{2(9.81) (9.27 \times 4)^2} = 9.64 \text{ m}$
 $\Delta E = 20.08 - 9.64 = 10.44 \text{ m}$

2.14. The discharge in a 20-m wide, rectangular, horizontal channel is $80 \text{ m}^3/\text{s}$ at a flow depth of 0.5 m upstream of a hydraulic jump. Determine the flow depth downstream of the jump and the head losses in the jump.

(2.14) / $B = 20 \text{ m}$ $Q = 80 \text{ m}^3/\text{s}$ $y_2 = ?$ $\Delta E = ?$

$\frac{y_2}{y_1} = \frac{1}{2} (-1 + \sqrt{1 + 8 Fr_1^2})$

$Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{Q}{A_1 \sqrt{g y_1}} = \frac{80}{(0.5 \times 20) \sqrt{9.81 \times 0.5}}$

$Fr_1 = 3.612$

$y_2 = \frac{1}{2} (0.5) (-1 + \sqrt{1 + 8(3.612)^2})$

$y_2 = 2.316 \text{ m}$

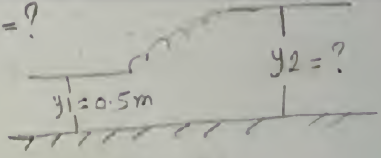
$E_1 = 0.5 + \frac{80^2}{2(9.81)(0.5 \times 20)^2} = 3.762 \text{ m}$

$E_2 = 2.316 + \frac{80^2}{2(9.81)(2.316 \times 20)^2} = 2.468 \text{ m}$

$\Delta E = 3.762 - 2.468$

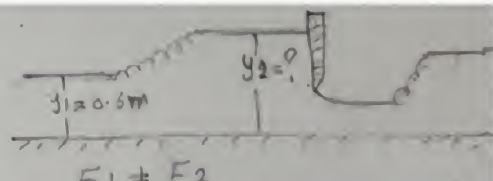
$\Delta E = 1.294 \text{ m}$

$E_1 \neq E_2$
 $F_{s1} = F_{s2}$



2.15. A 8-m wide rectangular channel is carrying a flow of $54 \text{ m}^3/\text{s}$ at a flow depth of 0.6 m. A sluice gate located at the downstream end of the channel controls the flow depth y_2 . Determine y_2 so that a hydraulic jump is formed upstream of the gate.

$B = 8 \text{ m}$ $Q = 54 \text{ m}^3/\text{s}$ $y_1 = 0.6 \text{ m}$
 $\frac{y_2}{y_1} = \frac{1}{2} (-1 + \sqrt{1 + 8 Fr_1^2})$
 $Fr_1 = \frac{V}{\sqrt{gy_1}} = \frac{Q}{A_1 \sqrt{gy_1}} = \frac{54}{(0.6 \times 8) \sqrt{9.81 \times 0.6}}$
 $Fr_1 = 4.637$
 $y_2 = \frac{1}{2} (0.6) (-1 + \sqrt{1 + 8 (4.637)^2})$
 $y_2 = 3.646 \text{ m}$



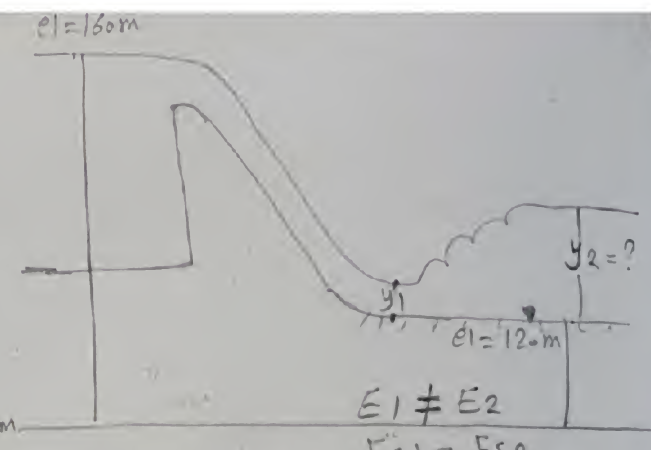
2.19. Determine the required depth in the river downstream of a 120-m wide spillway to form a hydraulic jump at its toe for the following data. The upstream reservoir level is at El. 160 m, the spillway discharge is $1200 \text{ m}^3/\text{s}$ and the river bottom level is at El. 120. Assume the losses on the spillway face are negligible and the stilling basin walls are vertical. Compute the energy losses in the jump.

2.19 $y_2 = ?$ $B = 120 \text{ m}$
 $\Delta E = ?$ $Q = 1200 \text{ m}^3/\text{s}$

$H_1 = H_2$
 $160 = y_1 + 120 + \frac{V_1^2}{2g}$
 $Q = V_1 y_1 B$
 $V_1 = \frac{Q}{y_1 B} = \frac{1200}{y_1 (120)}$
 $V_1 = \frac{10}{y_1}$

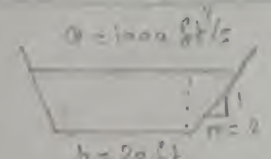
$160 - y_1 - 120 - \frac{100}{2(9.81)y_1^2} = 0$
 $40 - y_1 - \frac{100}{y_1^2 (19.62)} = 0$
 $40 = y_1 + \frac{100}{y_1^2 (19.62)}$
 $y_1 = 0.36 \text{ m}$
 $y_c = \left(\frac{q^2}{g} \right)^{1/3}$
 $y_c = \left(\frac{100}{9.81} \right)^{1/3} = 2.17 \text{ m}$
 $y_1 < y_c \therefore \checkmark$

$q = \frac{1200}{120} = 10 \text{ m}^2/\text{s}$
 $\frac{y_2}{y_1} = \frac{1}{2} [-1 + \sqrt{1 + 8 Fr_1^2}]$
 $Fr_1 = \frac{V}{\sqrt{gy_1}} = \frac{Q}{A_1 \sqrt{gy_1}} = \frac{1200}{(0.36 \times 120) \sqrt{9.81 \times 0.36}}$
 $Fr_1 = 14.78 > 1$
 $y_2 = 0.5 (0.36) [-1 + \sqrt{1 + 8 (14.78)^2}]$
 $y_2 = 7.35 \text{ m}$
 $E_1 = y_1 + \frac{q^2}{2gy_1^2} = 39.69 \text{ m}$
 $E_2 = y_2 + \frac{q^2}{2gy_2^2} = 7.44 \text{ m}$
 $\Delta E = 32.25 \text{ m}$



- 3.1. A hydraulic jump is to be formed in a trapezoidal channel with a base width of 20 ft and side slopes of 2:1. The upstream depth is 1.25 ft and $Q = 1000$ cfs. Find the downstream depth and the head loss in the jump. Solve by Figure 3.2 and verify by manual calculations. Compare the results for the sequent depth ratio and relative head loss with those in a rectangular channel of the same bottom width and approach Froude number.

8.1 $Q = 1000 \text{ ft}^3/\text{s}$ $y_2 = ?$ $\Delta E = ?$



$b = 20 \text{ ft}$ $m = 2$

$y_1 = 1.25 \text{ ft}$ $y_2 = ?$

$$Z_{f_{mp2}} = \frac{Q}{m y_1^2 (3 y_1)^{0.5}} = \frac{1000}{2 (1.25)^2 (32.2 \times 1.25)^{0.5}} = 50.44 \rightarrow X$$

$$\frac{b}{m y_1} = \frac{20}{2 (1.25)} = 8$$

$$\frac{y_2}{y_1} = 6.4 \Rightarrow y_2 = 6.4 \times (1.25) = 8 \text{ ft}$$

or by trial and error

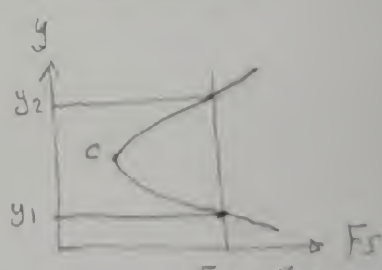
$$F_{s1} = \frac{Q^2}{8 A_1} + \bar{Z}_1 A_1$$

$$\bar{Z}_1 A_1 = \frac{y_1^2}{6} (2 m y_1 + 3 b)$$

$$\bar{Z}_1 A_1 = \frac{1.25^2}{6} (2(2)(1.25) + 3(20))$$

$$\bar{Z}_1 A_1 = 16.927$$

$$F_{s1} = \frac{1000^2}{32.2 (28.125)} + 16.927$$

$$F_{s1} = 1121.14 \text{ ft}^3$$


$F_{s1} = F_{s2}$

$$A_1 = b y_1 + m y_1^2 = 28.125 \text{ ft}^2$$

$$A_2 = b y_2 + m y_2^2$$

$$1121.14 = \frac{1000^2}{32.2 (20 y_2 + 2 y_2^2)} + \frac{y_2^2}{6} (4 y_2 + 3(20))$$

$$y_2 = 8.12 \text{ ft}$$

$$\Delta E = E_1 - E_2$$

$$E_1 = y_1 + \frac{Q^2}{A_1^2 2g} = 1.25 + \frac{1000^2}{2 (32.2) (28.125)^2} = 20.88 \text{ ft}$$

$$E_2 = y_2 + \frac{Q^2}{2g A_2^2} = 8.12 + \frac{1000^2}{2 (32.2) (294.27)^2} = 8.30 \text{ ft}$$

$$\therefore \Delta E = 12.58 \text{ ft}$$

Example 1) Determine the momentum function for the following three non-rectangular settings:

- A trapezoidal cross-section with $b = 5.0$ m, $m = 2$, $Q = 10$ m³/s, and $y = 0.5$ m.
- A trapezoidal cross-section with $b = 5.0$ m, $m = 2$, $Q = 10$ m³/s, and $y = 2.0$ m.
- A circular cross-section with $D = 5.0$ m, $Q = 10$ m³/s, and $y = 0.5$ m.

Solution:

- The cross-sectional area of a trapezoidal cross-section for the geometry given is calculated as

$$A = by + my^2 = 5 \cdot (0.5) + 2 \cdot (0.5)^2 = 3.0 \text{ m}^2$$

The momentum function then gives

$$\begin{aligned} M &= A\bar{y} + \frac{Q^2}{gA} = \frac{y^2}{6}(2my + 3b) + \frac{Q^2}{gA} \\ &= \frac{(0.5)^2}{6}(2 \cdot 2 \cdot (0.5) + 3 \cdot 5) + \frac{(10)^2}{(9.81) \cdot (3.0)} = 4.11 \text{ m}^3 \end{aligned}$$

- Similar to part (a), the cross-sectional area is

$$A = 5 \cdot (2.0) + 2 \cdot (2.0)^2 = 18.0 \text{ m}^2$$

The momentum function then gives

$$M = \frac{(2.0)^2}{6}(2 \cdot 2 \cdot (2.0) + 3 \cdot 5) + \frac{(10)^2}{(9.81) \cdot (18.0)} = 15.9 \text{ m}^3$$

- The area of a circular cross-section for the geometry given is calculated using the Appendix of this book. For the given problem we have

$$\frac{y}{D} = \frac{0.5}{5.0} = 0.1$$

Entering the Appendix for $y/D = 0.1$, we find that $A/D^2 = 0.0409$, and $\bar{y}/D = 0.0404$. Thus,

$$A = \frac{A}{D^2} \cdot D^2 = 0.0409 \cdot (5.0)^2 = 1.0225 \text{ m}^2$$

and

$$\bar{y} = \frac{\bar{y}}{D} \cdot D = 0.0404 \cdot (5.0) = 0.202 \text{ m}$$

Thus, the momentum function becomes

$$M = A\bar{y} + \frac{Q^2}{gA} = (1.0225) \cdot (0.202) + \frac{(10)^2}{(9.81) \cdot (1.0225)} = 10.2 \text{ m}^3$$

Example 2) use the charts to determine the conjugate depth of the hydraulic section in the following non-rectangular sections:

- A trapezoidal cross-section with $b = 5.0$ m, $m = 2$, $Q = 10$ m³/s, and $y = 0.5$ m.
- A trapezoidal cross-section with $b = 5.0$ m, $m = 2$, $Q = 10$ m³/s, and $y = 2.0$ m.
- A circular cross-section with $D = 5.0$ m, $Q = 10$ m³/s, and $y = 0.5$ m.

Solution:

- First we calculate the Froude number:

$$F_r = \frac{Q}{A \sqrt{g \cdot \left(\frac{A}{B} \right)}} = \frac{10}{3.0 \sqrt{9.81 \cdot \left(\frac{3.0}{7.0} \right)}} = 1.63$$

Since the Froude number is greater than 1, the flow is supercritical so the depth provided must be the upstream depth, y_1 . Using Figure 3.10, we note that we have all the values to calculate the horizontal axis quantity, Z :

$$Z = \frac{Q}{m y_1^2 \sqrt{g y_1}} = \frac{10}{2 \cdot (0.5)^2 \sqrt{(9.81) \cdot (0.5)}} = 9.03$$

In addition, we need to calculate the channel geometry factor:

$$\frac{b}{m y_1} = \frac{5}{2 \cdot 0.5} = 5.0$$

Entering the Figure of trapezoidal section on the horizontal axis at $Z = 9.03$ (use 9), we trace vertically up to the channel geometry curve corresponding to $b/m y_1 = 5$ and then move horizontally, pulling off the value $Y_2/Y_1 = 1.9$. (This is an approximate estimate.) Solving for Y_2 , we get

$$y_2 = \frac{y_2}{y_1} \cdot y_1 = (1.9) \cdot (0.5) = 0.95 \text{ m}$$

Using $y_2 = 0.95$, we first calculate the area:

$$A = by + my^2 = 5 \cdot (0.95) + 2 \cdot (0.95)^2 = 6.555 \text{ m}^2$$

The momentum function then gives

$$M = \frac{y^2}{6}(2my + 3b) + \frac{Q^2}{gA} = \frac{(0.95)^2}{6}(2 \cdot 2 \cdot (0.95) + 3 \cdot 5) + \frac{(10)^2}{(9.81) \cdot (6.555)} = 4.38 \text{ m}^3$$

The percent error, E , in estimating the momentum function is thus,

$$E = \frac{(4.38 - 4.11)}{4.11} \cdot 100\% = 6.6\%$$

b. First we calculate the Froude number:

$$F_r = \frac{Q}{A \sqrt{g \cdot \left(\frac{A}{B} \right)}} = \frac{10}{18.0 \sqrt{9.81 \cdot \left(\frac{18.0}{13.0} \right)}} = 0.15$$

Since the Froude number is less than 1, the flow is *subcritical*, so the depth provided must be the *downstream* depth, Y_2 . Note that Figures trapezoidal and circular sections are structured in terms of the upstream depth, Y_1 being known. That is not the case with this example. In order to use Figure of trapezoidal section, we need to guess a value for Y_1 , solve for Y_2 , and then see how the actual value of Y_2 compares with the calculated value. If necessary, iteration of this approach is indicated until the actual and calculated values of Y_2 are essentially equal. Looking at part (a) for guidance, we see that the conjugate depth pair is (0.5 m, 0.95 m). Since Y_2 in part (b) is more than twice as big as in part (a), we guess an initial Y_1 that is only half as big as Y_1 in part (a), thus we guess $Y_1 = 0.25$ m. So,

$$Z = \frac{10}{2 \cdot (0.25)^2 \sqrt{(9.81) \cdot (0.25)}} = 51.1$$

The channel geometry factor is

$$\frac{b}{my_1} = \frac{5}{2 \cdot (0.25)} = 10$$

Entering Figure of trapezoidal section on the horizontal axis at $Z = 51.1$ (use approximately 50), we trace vertically up to the channel geometry curve corresponding to $b/my_1 = 10$ and then move horizontally pulling off the value $Y_2/Y_1 = 5.7$. (This is an approximate estimate.) Solving for Y_2 , we get

$$y_2 = \frac{Y_2}{Y_1} \cdot y_1 = (5.7) \cdot (0.25) = 1.42 \text{ m}$$

Since $1.42 \text{ m} < 2.0 \text{ m}$, we know that our initial guess for y_1 was too deep. We need to make a new estimate of y_1 smaller than the original guess of 0.25 m . Try $y_1 = 0.10 \text{ m}$. Thus $Z = 504$, the channel geometry factor is 25, and $y_2/y_1 = 24$ (approximately), leading to a y_2 estimate of 2.4 m . So our second guess was too shallow, but now we have the solution bounded. Iteration continues in this fashion. Make a final guess for y_1 of 0.12 . Thus, $Z = 320$, and the channel geometry factor is 20.8, leading to $y_2/y_1 = 17$ (approximately), leading to a y_2 estimate of 2.04 m (close enough to 2.0 m).

The momentum function then gives

$$M = \frac{(2.04)^2}{6} (2 \cdot 2 \cdot (2.04) + 3 \cdot 5) + \frac{(10)^2}{(9.81) \cdot (18.523)} = 16.6 \text{ m}^3$$

The percent error, E , is thus

$$E = \frac{(16.6 - 15.9)}{15.9} \cdot 100\% = 4.4\%$$

c- using the figure of circular section, we calculate the horizontal axis quantity:

$$Z = \frac{Q^2}{gy_1^5} = \frac{(10)^2}{(9.81) \cdot (0.5)^5} = 326$$

The channel geometry factor is

$$\frac{y_1}{D} = \frac{0.5}{5.0} = 0.1$$

Entering Figure 3.11 on the horizontal axis at $Z = 326$ (use best visual approximation of $Z = 326$), we trace vertically up to the channel geometry curve corresponding to $y_1/D = 0.1$, and then move horizontally, pulling off the value $y_1/y_2 = 0.21$. (This is an approximate estimate.) Solving for y_2 , we get

$$y_2 = \frac{1}{y_1/y_2} \cdot y_1 = \frac{0.5}{0.21} = 2.38 \text{ m}$$

Exercise 1) A circular culvert forms the outlet structure of a lake that discharges to a steep channel. Local design codes require that the depth at the outfall from the lake be no more than 0.4 m for a discharge of 0.75 m³/s. What is the smallest diameter culvert that will satisfy the design codes?

Solution: The code focuses on the depth of flow at the outlet. So, the depth of 0.4 m is the critical depth. Thus, using the figure below, we can determine the diameter of the culvert.

The solution begins by determining $M_{design, circular}$:

$$M_{design, circular} = \frac{Q}{y_c^2 \sqrt{gy_c}} = \frac{0.75}{(0.4)^2 \cdot \sqrt{(9.81) \cdot (0.4)}} = 2.4$$

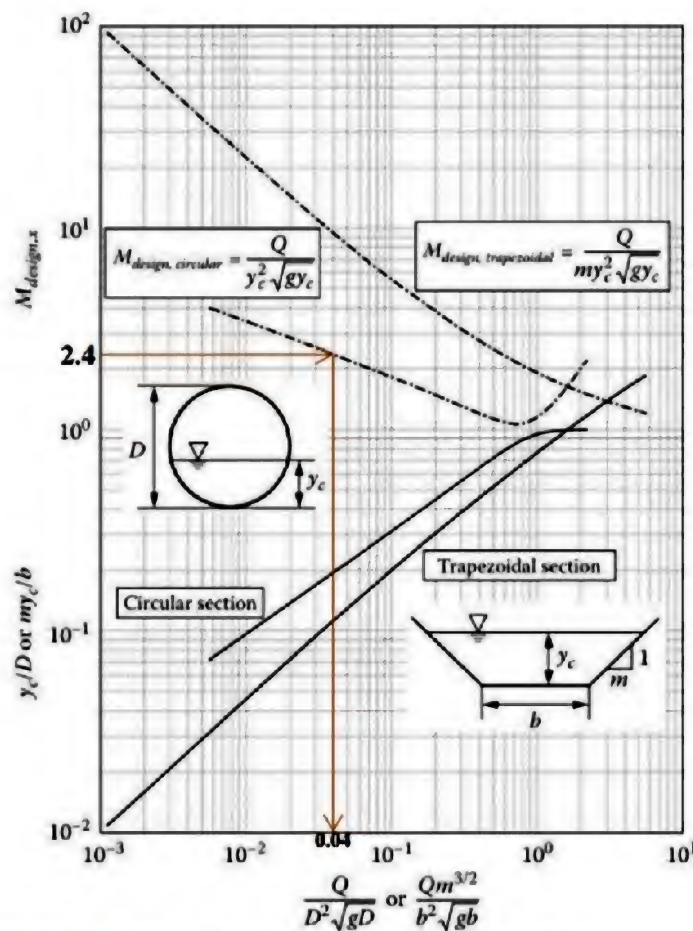


FIGURE 2.24 Critical depth, y_c , in trapezoidal and circular cross-sections.

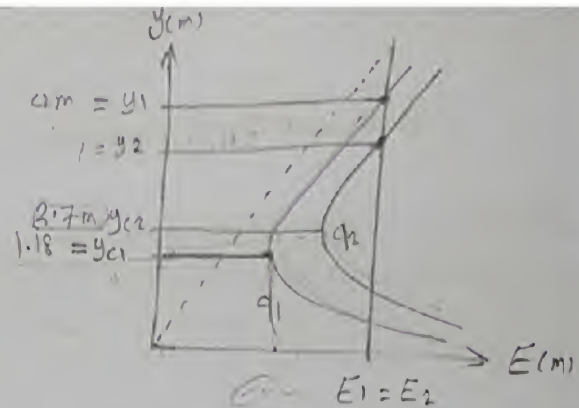
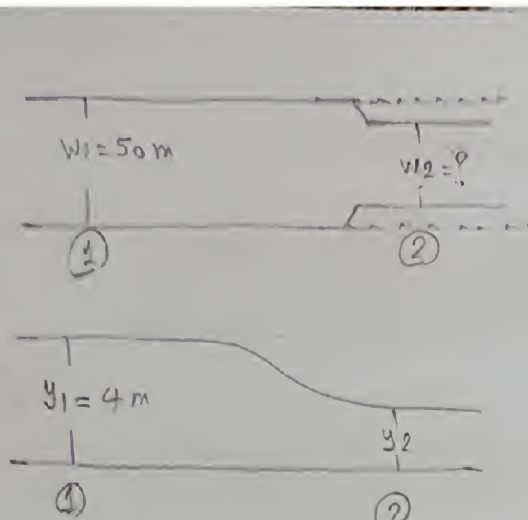
Using the horizontal axis value, we get

$$0.04 = \frac{Q}{D^2 \sqrt{gD}}$$

Rearranging and solving for pipe diameter, D ,

$$D = \left(\frac{Q}{(0.04) \cdot \sqrt{g}} \right)^{2/5} = \left(\frac{0.75}{(0.04) \cdot \sqrt{9.81}} \right)^{2/5} = 2.0 \text{ m}$$

Exercise 2) A bridge is planned on a 50-m wide rectangular channel carrying a flow of $200 \text{ m}^3/\text{s}$ at a flow depth of 4.0 m. For reducing the length of the bridge, what is the minimum channel width such that the upstream water level is not influenced for this discharge?



$$Q = 200 \text{ m}^3/\text{s}$$

$$q = \frac{Q}{b}$$

$$W_1 > W_2$$

$$q_1 < q_2$$

$$q_1 = \frac{200}{50} = 4 \text{ m}^2/\text{s}$$

$$E_1 = y_1 + \frac{q_1^2}{2gy_1^2} = 4 + \frac{4^2}{2(9.81)(4)^2}$$

$$E_1 = 4.05 \text{ m}$$

$$W_2 = \frac{Q}{q} \quad \text{--- (1)}$$

$$q = \sqrt{y_c^3 g}$$

$$y_c = \frac{2}{3} E_1$$

$$y_c = \frac{2}{3} (4.05) = 2.7 \text{ m}$$

$$W_2 = \frac{200}{\sqrt{(2.7)^3 \cdot 9.81}} = 14.39 \text{ m}$$

$$q_2 = \frac{200}{14.39} = 13.9 \text{ m}^2/\text{s}$$

$$y_{c,2} = \left(\frac{q_2^2}{g} \right)^{1/3}$$

$$y_{c,2} = \left(\frac{13.9^2}{9.81} \right)^{1/3} = 2.7 \text{ m}$$

$$E_{c,2} = \frac{3}{2} y_{c,2}$$

$$E_{c,2} = 4.05 \text{ m}$$

$$\therefore E_1 = E_{c,2}$$

Exercise 3) A 2.5-m wide rectangular channel carries $6.0 \text{ m}^3/\text{s}$ of flow at a depth of 0.50 m. Calculate the minimum height of a streamlined, flat-topped hump required to be placed at a section to cause critical flow over the hump. The energy loss over the hump can be taken as 10% of the upstream velocity head.

The diagram shows a rectangular channel of width $B_1 = B_2 = 2.5 \text{ m}$. At section 1 (upstream), the depth is $y_1 = 0.5 \text{ m}$. At section 2 (over the hump), the depth is $y_2 = y_c$. The hump height is Z . Section 3 is downstream. The flow is from left to right.

Given data:

- $B = 2.5 \text{ m}$
- $Q = 6 \text{ m}^3/\text{s}$
- $y = 0.5 \text{ m}$
- $h_L(2) = 10\% \frac{V_2^2}{2g}$

Calculations:

$$q_1 = q_2 = q = \frac{Q}{B} = \frac{6}{2.5} = 2.4 \text{ m}^2/\text{s}$$

$$E_1 = y_1 + \frac{q^2}{2gy_1^2}$$

$$E_1 = 0.5 + \frac{(2.4)^2}{2(9.81)(0.5)^2}$$

$$E_1 = 1.67 \text{ m}$$

$$V_2 = \sqrt{gy_c}$$

$$y_c = \left(\frac{q^2}{g} \right)^{1/3}$$

$$y_c = \left(\frac{2.4^2}{9.81} \right)^{1/3} = 0.84 \text{ m}$$

$$h_L(2) = 0.1 \times \frac{q^2}{2gy_1^2}$$

$$h_L(2) = 0.1 \times \frac{2.4^2}{2(9.81)(0.5)^2}$$

$$h_L(2) = 0.117 \text{ m}$$

$Z = ?$

$$E_1 = E_2 + Z + h_L$$

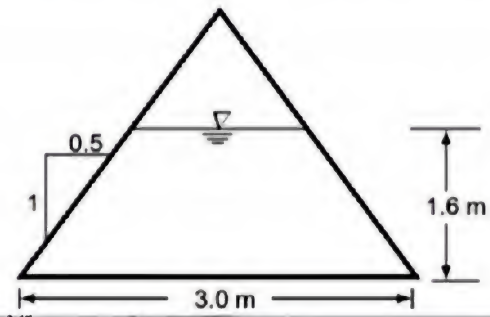
$$E_2 = y_c + \frac{q^2}{2gy_c^2} = 0.84 + \frac{2.4^2}{2(9.81)(0.84)^2} = 1.256 \text{ m}$$

$$1.67 = 1.256 + Z + 0.117$$

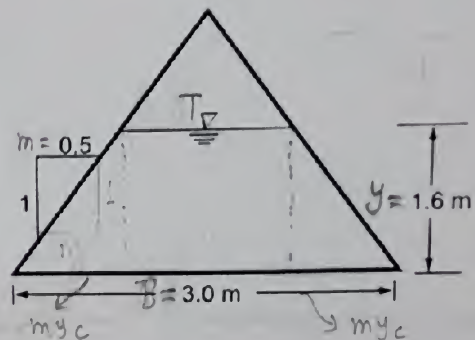
$$Z = 1.67 - 1.256 - 0.117$$

$$Z = 0.30 \text{ m}$$

Exercise 4) Water is flowing at a critical depth at a section in a Δ shaped channel, with side slope of 0.5 H: 1 V. (Fig. 2.8). If the critical depth is 1.6 m, estimate the discharge in the channel and the specific energy at the critical depth section.



$Q = ?$ $E = ?$ $y_c = 1.6 \text{ m}$
 E_{\min}



$$E = y_c + \frac{V^2}{2g} = y_c + \frac{Q^2}{2g A^2} \quad (1)$$

$$T = B - 2m y_c$$

$$T = 3 - 2(0.5)(1.6)$$

$$T = 1.4 \text{ m}$$

$$A = (1.4 \times 1.6) + (0.5)(1.6)^2$$

$$A = 3.52 \text{ m}^2$$

$$\therefore Fr = \frac{V}{\sqrt{g y}} = \frac{Q}{A \sqrt{g \frac{A}{T}}}$$

$$1 = \frac{Q}{3.52 \sqrt{\frac{9.81 \times 3.52}{1.4}}}$$

$$Q = 17.48 \text{ m}^3/\text{s}$$

$$E = y + \frac{V^2}{2g}$$

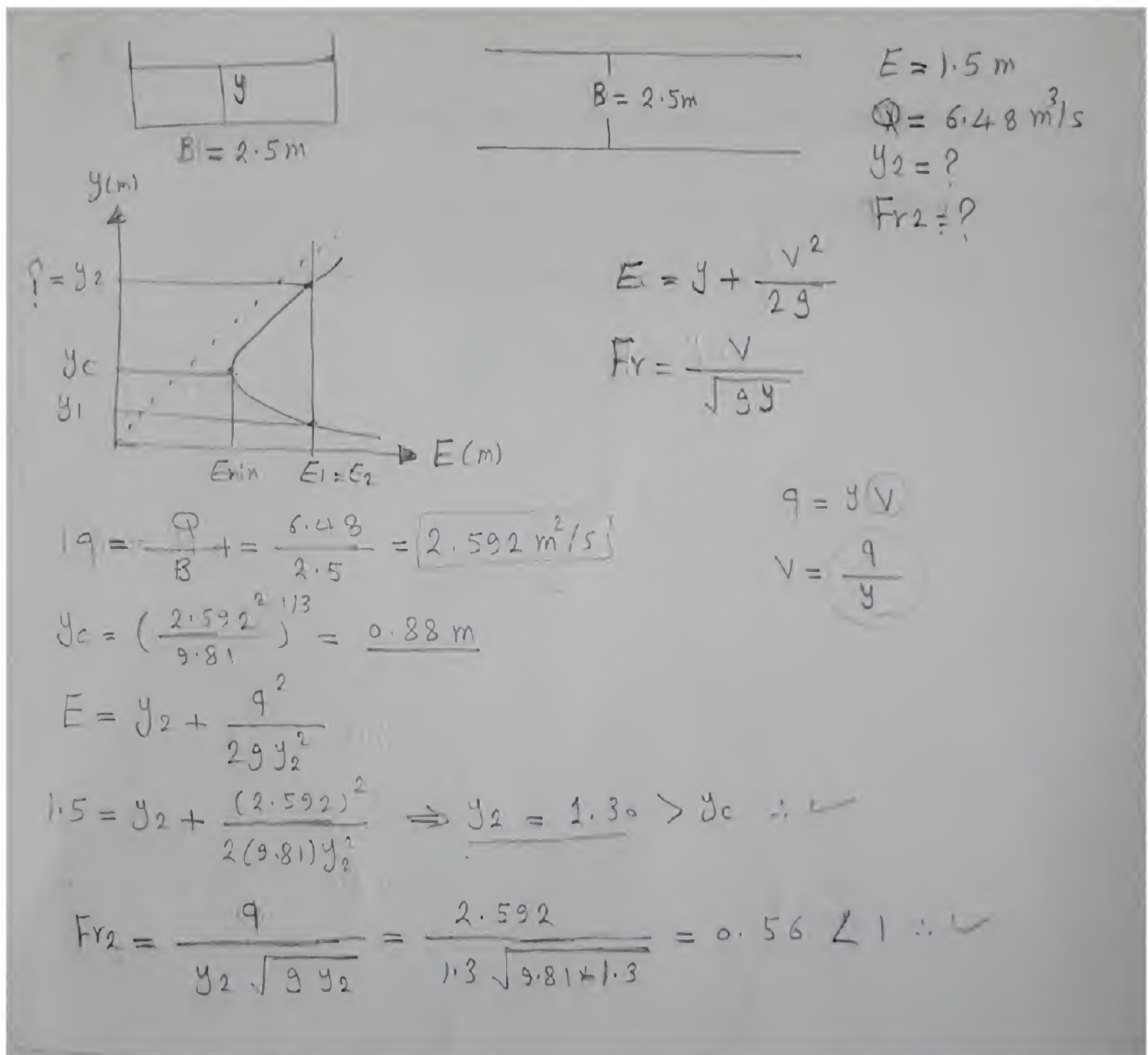
$$E = y + \frac{Q^2}{A^2 2g}$$

$$E = 1.6 + \frac{(17.48)^2}{(3.52)^2 2(9.81)}$$

$$E = 2.86 \text{ m}$$

Exercise 5) A 2.5-m wide rectangular channel has a specific energy of 1.50 m when carrying a discharge of 6.48 m³/s. Calculate the alternate depths and corresponding Froude numbers.

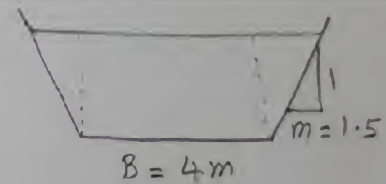
Solution:



Exercise 6) A trapezoidal channel with a bed width of 4.0 m and side slopes of 1.5 H: 1 V carries a certain discharge. (a) Based on observations, if the critical depth of the flow is estimated as 1.70 m, calculate the discharge in the channel. (b) If this discharge is observed to be flowing at a depth of 2.50 m in a reach, estimate the Froude number of the flow in that reach.

a) $y_c = 1.7 \text{ m}$ $Q = ?$

b) $Q = ?$ $y = 2.5 \text{ m}$ $Fr = ?$



a) $y = y_c = 1.7 \text{ m} \therefore Fr = 1$

$Q = ?$

$$A = (4 + 1.7) + 1.5(1.7)^2$$

$$A = 11.135 \text{ m}^2$$

$$T = 4 + 2(1.5)(1.7)$$

$$T = 9.1 \text{ m}$$

$$Fr = \frac{V}{\sqrt{gy}} = \frac{Q}{A \sqrt{g \frac{A}{T}}}$$

$$1 = \frac{Q}{11.135 \sqrt{\frac{9.81 \times 11.135}{9.1}}}$$

$$Q = 38.58 \text{ m}^3/\text{s}$$

b) $Q = 38.58 \text{ m}^3/\text{s}$

$y = 2.5$ $Fr = ?$

$$Fr = \frac{V}{\sqrt{gy}} = \frac{Q}{A \sqrt{g \frac{A}{T}}} \quad \text{--- (1)}$$

$$A = (4 \times 2.5) + 1.5(2.5)^2$$

$$A = 19.375 \text{ m}^2$$

$$T = 4 + 2(1.5)(2.5)$$

$$T = 11.5 \text{ m}$$

$$\therefore Fr = \frac{38.58}{19.375 \sqrt{\frac{9.81 \times 19.375}{11.5}}}$$

$$Fr = 0.49 < 1 \therefore \checkmark$$

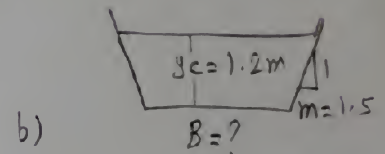
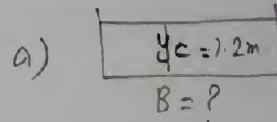
Exercise 7) Calculate the bottom width of a channel required to carry a discharge of $15.0 \text{ m}^3/\text{s}$ as a critical flow at a depth of 1.2 m , if the channel section is (a) rectangular, and (b) trapezoidal with side slope 1.5 horizontal: 1 vertical.

$$Q = 15 \text{ m}^3/\text{s} \quad y_c = 1.2 \text{ m} = y$$

a) rectangular $\Rightarrow B = ?$

b) trapezoidal $\Rightarrow B = ?$

$$H = 1.5 : V = 1$$



$$\begin{aligned} \text{a) } y_c &= \left(\frac{Q^2}{g} \right)^{1/3} \Rightarrow y_c^3 = \frac{Q^2}{g} \Rightarrow y_c^3 = \frac{Q^2}{B^2 g} \Rightarrow y_c^3 = \frac{Q^2}{g B^2} \\ Q^2 &= y_c^3 g B^2 \Rightarrow B^2 = \frac{Q^2}{y_c^3 g} = \frac{15^2}{(1.2)^3 \cdot 9.81} \\ B &= 3.64 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{b) } A &= By + my^2 & T &= B + 2(1.5)(1.2) \\ A &= 1.2B + (1.5)(1.2)^2 & T &= B + 3.6 \\ A &= 1.2B + 2.16 \end{aligned}$$

$$\begin{aligned} Q &= AV & Fr &= \frac{V}{\sqrt{gy}} = \frac{V}{\sqrt{g \frac{A}{T}}} \\ V &= \sqrt{9.81 \frac{1.2B + 2.16}{B + 3.6}} \\ 1.5 &= (1.2B + 2.16) \sqrt{9.81 \left(\frac{1.2B + 2.16}{B + 3.6} \right)} \\ B &= 2.53 \text{ m} \end{aligned}$$

Chapter Four

Problems

Compute the normal depth in a trapezoidal channel having a bottom-width of 10 m, side slopes of 2H to 1V and carrying a flow of $30 \text{ m}^3/\text{s}$. The slope of the channel bottom is 0.001 and $n = 0.013$.

1// $y_n = ?$ $Q = 30 \text{ m}^3/\text{s}$
 $S_0 = 0.001$
 $n = 0.013$

$$Q = \frac{1}{n} A R_h^{2/3} S_f^{1/2}$$

$$A = b y_n + m y_n^2$$

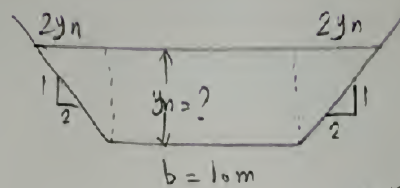
$$A = 10 y_n + 2 y_n^2 \quad \dots (1)$$

$$P = b + 2 \sqrt{4 y_n^2 + y_n^2}$$

$$P = b + 2 y_n \sqrt{5}$$

$$P = 10 + 2\sqrt{5} y_n \quad \dots (2)$$

$$R_h = \frac{A}{P} \Rightarrow R_h = \frac{A^{2/3}}{P^{2/3}}$$



$$Q = \frac{A}{n} \frac{A^{2/3} S_f^{1/2}}{P^{2/3}} = \frac{A^{5/3} S_f^{1/2}}{n P^{2/3}}$$

$$\frac{A^{5/3}}{P^{2/3}} = \frac{n Q}{S_f^{1/2}}$$

$$\frac{(10 y_n + 2 y_n^2)^{5/3}}{(10 + 2\sqrt{5} y_n)^{2/3}} = \frac{0.013 \times 30}{\sqrt{0.001}}$$

$$y_n = 1.09 \text{ m}$$

4.1 A trapezoidal channel is 10.0 m wide and has a side slope of 1.5 horizontal: 1 vertical. The bed slope is 0.0003. The channel is lined with smooth concrete of $n = 0.012$. Compute the mean velocity and discharge for a depth of flow of 3.0 m.

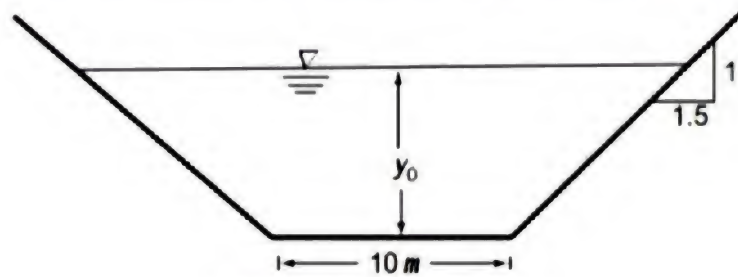


Fig. 3.11 Example 3.4

Solution Let y_0 = uniform flow depth
 Here $B = 10.0$ m and side slope $m = 1.5$
 Area $A = (B + my) y$
 $= (10.0 + 1.5 \times 3.0) 3.0 = 43.50 \text{ m}^2$

Wetted perimeter $P = B + 2y \sqrt{m^2 + 1}$
 $= 10.0 + 2 \sqrt{2.25 + 1} \times 3.0 = 20.817 \text{ m}$

Hydraulic radius $R = \frac{A}{P} = 2.090 \text{ m}$

Mean velocity $V = \frac{1}{n} R^{2/3} S_0^{1/2}$
 $= \frac{1}{0.012} \times (2.09)^{2/3} \times (0.0003)^{1/2}$
 $= 2.36 \text{ m/s}$

Discharge $Q = AV = 102.63 \text{ m}^3/\text{s}$

4.2 In the channel of Example 3.4, find the bottom slope necessary to carry only $50 \text{ m}^3/\text{s}$ of the discharge at a depth of 3.0 m .

Solution $A = 43.50 \text{ m}^2$

$$P = 20.817 \text{ m}$$

$$R = 2.09 \text{ m}$$

$$S_0 = \frac{Q^2 n^2}{A^2 R^{4/3}} = \frac{(50.0)^2 \times (0.012)^2}{(43.5)^2 \times (2.09)^{4/3}}$$

$$= 0.0000712$$

4.3 A 5.0-m wide trapezoidal channel having a side slope of 1.5 horizontal: 1 vertical is laid on a slope of 0.00035 . The roughness coefficient $n = 0.015$. Find the normal depth for a discharge of $20 \text{ m}^3/\text{s}$ through this channel.

Solution Let

$y_0 = \text{normal depth}$

Area $A = (5.0 + 1.5 y_0) y_0$

Wetted perimeter $P = 5.0 + 2 \sqrt{3.25} y_0$
 $= 5.0 + 3.606 y_0$

$$R = A/P = \frac{(5.0 + 1.5 y_0) y_0}{(5.0 + 3.606 y_0)}$$

The section factor $AR^{2/3} = \frac{Qn}{\sqrt{S_0}}$

$$\frac{(5.0 + 1.5 y_0)^{5/3} y_0^{5/3}}{(5.0 + 3.606 y_0)^{2/3}} = \frac{20 \times 0.015}{(0.00035)^{1/2}} = 16.036$$

Algebraically, y_0 can be found from the above equation by the trial-and-error method. The normal depth is found to be 1.820 m

4.4

A concrete-lined trapezoidal channel ($n = 0.015$) is to have a side slope of 1.0 horizontal: 1 vertical. The bottom slope is to be 0.0004. Find the bottom width of the channel necessary to carry 100 m³/s of discharge at a normal depth of 2.50 m.

Solution Let B = bottom width. Here, y_0 = normal depth = 2.50 m, $m = 1.0$

Area $A = (B + 2.5) \times 2.5$

Wetted perimeter $P = (B + 2\sqrt{2} \times 2.5) = B + 7.071$

$$\frac{Qn}{\sqrt{S_0}} = \frac{100 \times 0.015}{\sqrt{0.0004}} = 75 = AR^{2/3}$$

$$\frac{[(B + 2.5) \times 2.5]^{5/3}}{(B + 7.071)^{2/3}} = 75.0$$

By trial-and-error $B = 16.33 \text{ m.}$

4.5

A standard lined trapezoidal canal section is to be designed to convey 100 m³/s of flow. The side slopes are to be 1.5 horizontal: 1 vertical and Manning's $n = 0.016$. The longitudinal slope of the bed is 1 in 5000. If a bed width of 10.0 m is preferred what would be the normal depth?

Solution Referring to Fig. 3.16, m = side slope = 1.5

$$\varepsilon = m + \tan^{-1} \frac{1}{m} = 1.5 + \tan^{-1}(1/1.5) = 2.088$$

Further, here $Q = 100.0 \text{ m}^3/\text{s}$, $n = 0.016$, $S_0 = 0.0002$, $B = 10.0 \text{ m}$

$$\phi_1 = \frac{Qn\varepsilon^{5/3}}{S_0^{1/2}B^{8/3}} = \frac{100 \times 0.016 \times (2.088)^{5/3}}{(0.0002) \times (10.0)^{8/3}} = 0.8314$$

By Eq.(3.39) $\phi_1 = \frac{(1 + \eta_0)^{5/3} \eta_0^{5/3}}{(1 + 2\eta_0)^{2/3}} = 0.8314$

On Simplifying, $\frac{(1 + \eta_0)^{5/3} \eta_0^{5/3}}{(1 + 2\eta_0)^{2/3}} = 0.8951$

On solving by trial and error $\eta_0 = \frac{y_0 \varepsilon}{B} = 0.74$

The normal depth $y_0 = \frac{0.74 \times 10.0}{2.088} = 3.544 \text{ m}$

Chapter Five

Example 1: An open channel is made of concrete to be designed to carry $1.0 \text{ m}^3/\text{sec}$ at a slope of 0.0065 . Find the most efficient cross section for a semicircular section. Use $n = 0.011$

EX(1): $Q = 1 \text{ m}^3/\text{s}$ $S_0 = 0.0065$ $n = 0.011$

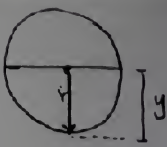
$A = \frac{\pi}{2} y^2$ $P = \pi y$

$R = \frac{A}{P} = \frac{\pi/2 y^2}{\pi y} = \frac{y}{2}$

$V = \frac{1}{n} R^{2/3} S_0^{1/2} \Rightarrow Q = \frac{S_0^{1/2}}{n} A R^{2/3}$

$1 = \frac{(0.0065)^{1/2}}{0.011} \left(\frac{\pi}{2}\right) y^2 \left(\frac{y}{2}\right)^{2/3} \Rightarrow 1 = \frac{(0.0065)^{1/2}}{0.011} \left(\frac{\pi}{2}\right) \left(\frac{1}{2}\right)^{2/3} y^{8/3}$

$\therefore y = 0.476 \text{ m}$



Example 2: A concrete open channel carrying a discharge of $1 \text{ m}^3/\text{sec}$ rectangular section at a bed slope 0.0065 . Use $n = 0.011$

EX(2): $Q = 1 \text{ m}^3/\text{s}$ $S_0 = 0.0065$ $n = 0.011$

$A = by \Rightarrow b = \frac{A}{y}$

$P = b + 2y$

$P = \frac{A}{y} + 2y, \frac{dP}{dy} = 0$

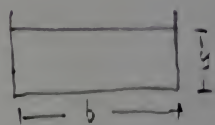
$0 = -\frac{A}{y^2} + 2 \Rightarrow \frac{A}{y^2} = 2 \Rightarrow A = 2y^2$

$b = 2y \therefore P = 4y$ $R = \frac{A}{P} = \frac{2y^2}{4y} = \frac{y}{2}$

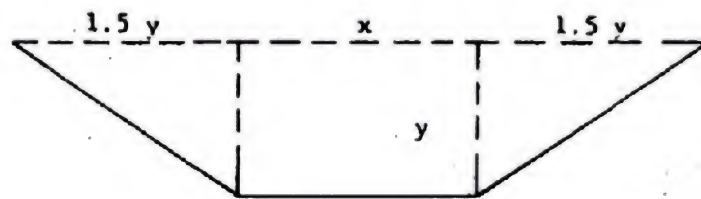
$Q = \frac{S_0^{1/2}}{n} A R^{2/3}$

$1 = \frac{(0.0065)^{1/2}}{0.011} 2y^2 \left(\frac{y}{2}\right)^{2/3} \Rightarrow 1 = \frac{(0.0065)^{0.5}}{0.011} (2) \left(\frac{1}{2}\right)^{2/3} y^{8/3}$

$\therefore y = 0.434 \text{ m} \therefore b = 2(0.434) = 0.868 \text{ m}$



Example 4: Determine the best hydraulic section for the trapezoidal channel of figure below, if the discharge is $10 \text{ m}^3/\text{sec}$ and the channel bottom slope is 0.0005 . Use $n = 0.02$.



$$A = by + 1.5y^2$$

$$A = y(b + 1.5y)$$

$$P = b + 2\sqrt{1.5^2y^2 + y^2}$$

$$P = b + 2y\sqrt{1.5^2 + 1}$$

$$P = b + \sqrt{13}y$$

$$\Rightarrow b + 1.5y = \frac{A}{y}$$

$$b = \left(\frac{A}{y} - 1.5y \right)$$

$$P = \frac{A}{y} - 1.5y + \sqrt{13}y, \quad \frac{\partial P}{\partial y} = 0$$

$$0 = -\frac{A}{y^2} - 1.5 + \sqrt{13} \Rightarrow -\frac{A}{y^2} = -(-1.5 + \sqrt{13})$$

$$\frac{A}{y^2} = \sqrt{13} - 1.5$$

$$A = (\sqrt{13} - 1.5)y^2$$

$$b = \frac{(\sqrt{13} - 1.5)y^2}{y} - 1.5y = (\sqrt{13} - 1.5)y - 1.5y$$

$$b = y(\sqrt{13} - 3)$$

$$A = by + 1.5y^2 = y^2(\sqrt{13} - 3) + 1.5y^2 = (\sqrt{13} - 1.5)y^2$$

$$P = y(\sqrt{13} - 3) + \sqrt{13}y = (\sqrt{13} - 3 + \sqrt{13})y = (2\sqrt{13} - 3)y$$

$$P = [2(\sqrt{13} - 1.5)]y$$

$$\therefore R = \frac{A}{P} = \frac{(\sqrt{13} - 1.5)y^2}{[2(\sqrt{13} - 1.5)]y}$$

$$R = \frac{(\sqrt{13} - 1.5)y}{2(\sqrt{13} - 1.5)}$$

$$R = \frac{y}{2}$$

$$Q = \frac{S_0^{1/2}}{n} A R^{2/3}$$

$$10 = \frac{(0.0005)^{0.5}}{0.02} (\sqrt{13} - 1.5)y^2 \left(\frac{y}{2} \right)^{2/3}$$

$$10 = \frac{(0.0005)^{0.5}}{0.02} (\sqrt{13} - 1.5) \left(\frac{1}{2} \right)^{2/3} y^{8/3}$$

$$\therefore y = 2.046 \text{ m}$$

$$b = 2.046(\sqrt{13} - 3)$$

$$b = 1.239 \text{ m}$$

Chapter Six

Example :

A rectangular channel has a width of 30 ft and a bed slope of 1:12100. The normal depth is 6 ft. A dam across the river elevates the water surface and produce a depth of 9.8 ft just upstream of dam. Find the length of backwater curve ; take $n = 0.025$?

Solution :

The discharge in the river can be calculated from the uniform flow condition.

$$A = 30 \times 6 = 180 \text{ ft}^2, p = 30 + 2 \times 6 = 42 \text{ ft}, R_h = A/p = 4.28 \text{ ft}$$

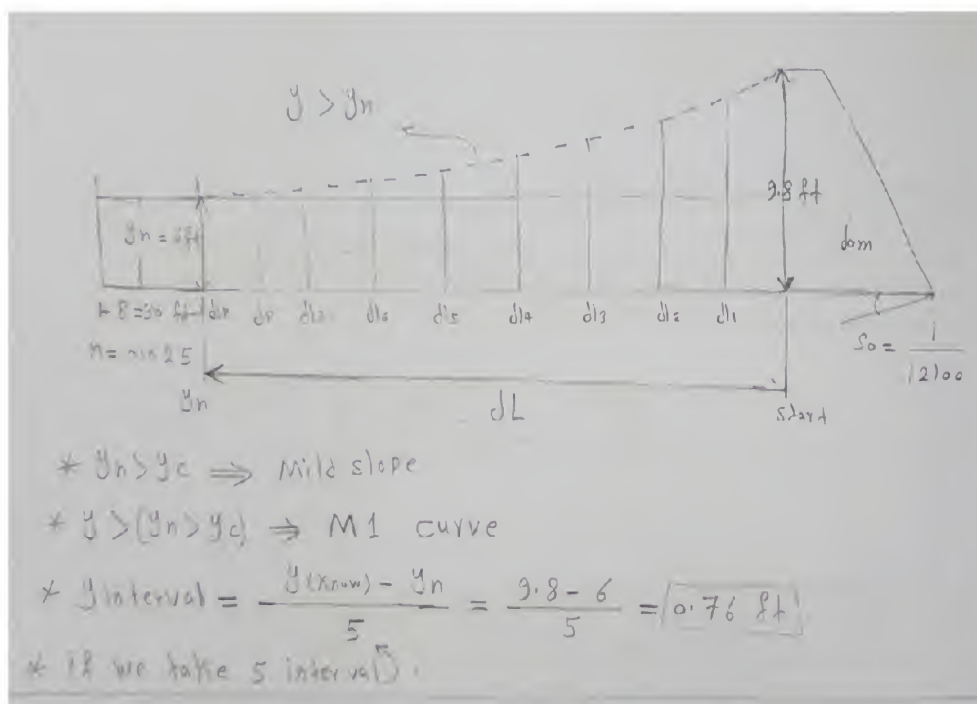
$$V = (1.49/n) \times R_h^{2/3} S_o^{1/2} = (1.49/0.025) \times (4.28)^{2/3} (1/12100)^{1/2}$$

$$V = 1.43 \text{ ft/sec}$$

$$Q = A \times v = 180 \times 1.43 = 257.4 \text{ ft}^3/\text{sec}$$

One needs to calculate the critical depth to determine the type of flow :

$$y_c = (8.58^2 / 32.2)^{1/3} = 1.31 \text{ ft}$$



y ft	A ft ²	V ft/sec	E ft	ΔE ft	R ft	S _r 10 ⁻⁵	S _r 10 ⁻⁵	(S _o - S _r) 10 ⁻⁵	dl ft
9.8	294	0.87	9.81	- 0.8	5.92	1.98	2.26	6.00	- 13333
9.0	270	0.95	9.01		5.62	2.54			
8.0	240	1.07	8.02	- 0.99	5.21	3.56	4.39	3.07	- 25840
				- 1.0					
7.0	210	1.22	7.02	- 0.99	4.77	5.22	6.78	1.52	- 65131
6.0	180	1.43	6.03						
Total									-123306 ft

Since $y_o > y_c$, the river has a mild bed slope and the M1 profile will be formed to elevate the water surface.

EXAMPLE:

A dam is built near the downstream end of a rectangular channel of 3.0 m width and 0.001 bed slope. The channel carries a discharge of 8.5 m³/sec with a normal depth of 1.5 m. If the depth of flow just before the dam is 2.5 m, how far upstream will the back water curve cause a velocity reduction of 20 % as compared to the velocity of the uniform flow ? Take the Chezy coefficient as 69 .

Solution:

The normal depth, y_o , is equal to 1.5 m.

Evaluate of , y_c , to know the category of bed slope .

Now $q = Q / B = 2.83 \text{ m}^3 / \text{sec.m.}$

$$y_c = (q^2 / g)^{1/3} = 0.935 \text{ m}$$

since $y_o > y_c$, then the bed has a mild slope and an M1 CURVE will be found before the dam.

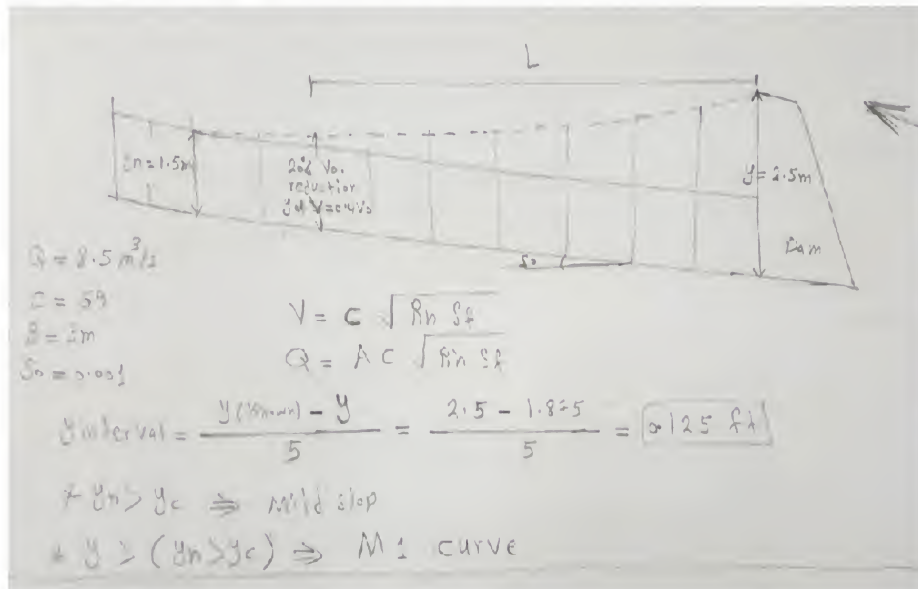
Applying the continuity eq. between a section of uniform flow and section where the velocity is reduced by 20 % ,

$$y_o * V_o = y * 0.8 V_o , y = y_o / 0.8 = 1.875 \text{ m}$$

It is now required to calculate the length of of the M1 profile between $y_1 = 2.5 \text{ m}$ and $y = 1.875 \text{ m}$.

Now consider two intermediate depths $y = 2.3 \text{ m}$ and $y = 2.1 \text{ m}$.

The computation start from the section at the dam and proceed upstream to compute the required length as shown in table below .



y m	A m ²	V m/sec	E m	ΔE m	R m	S _f 10 ⁻⁵	S _f 10 ⁻⁵	(S _o - S _f) 10 ⁻⁵	dl m
2.5	7.5	1.133	2.565		0.937	28.76			
2.3	6.9	1.232	2.377	- 0.188	0.907	35.11	31.93	68.1	_ 276
2.1	6.3	1.349	2.193	- 0.184	0.875	43.68	39.39	60.6	_ 303
1.875	5.6	1.511	1.991	- 0.202	0.833	57.57	50.62	49.4	_ 408
								Σ L =	_ 987 m

Exercise 1: A very wide rectangular channel carries a discharge of $10.0 \text{ m}^3/\text{s}/\text{m}$ on a slope of 0.001 with an n value of 0.026 . The channel ends in a free over fall. Compute the distance required for the depth to reach 0.9 of normal depth? Take 5 equal intervals for the computation of depths.

$q = 10 \text{ m}^2/\text{s} \rightarrow$

$S_0 = 0.001$
 $n = 0.026$

\therefore Rectangular channel
 $\therefore R_h = y$

$V = \frac{1}{n} R_h^{2/3} S_0^{1/2}$
 $Q = \frac{1}{n} A R_h^{2/3} S_0^{1/2}$
 $q = \frac{1}{n} \frac{A}{B} R_h^{2/3} S_0^{1/2}$
 $q = \frac{1}{n} y_n R_h^{2/3} S_0^{1/2}$
 $q = \frac{1}{n} y_n^{5/3} S_0^{1/2}$

$Q = AV$

$y_n = \left(\frac{q n}{S_0^{1/2}} \right)^{3/5}$
 $y_n = 3.54 \text{ m}$
 $y = 0.9 \times 3.54 = 3.186 \text{ m}$
 $y_c = \left(\frac{q^2}{g} \right)^{1/3} = \left(\frac{10^2}{9.81} \right)^{1/3} = 2.17 \text{ m}$

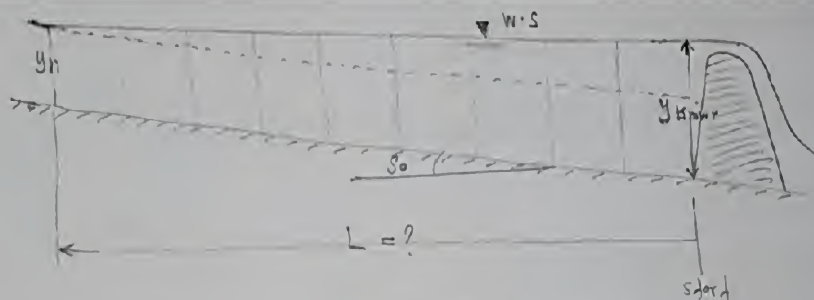
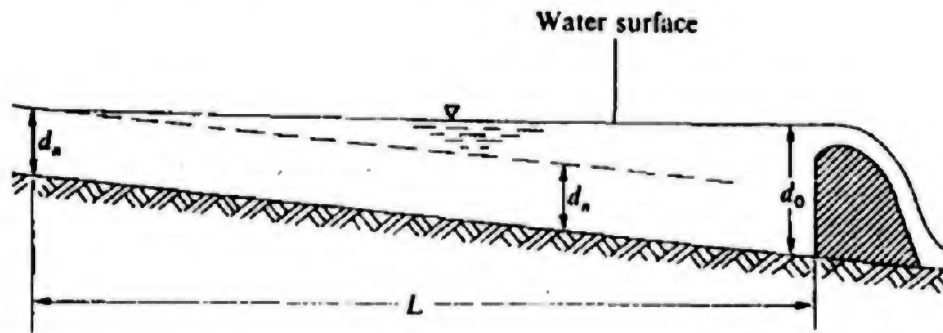
$\therefore y_n > y_c \Rightarrow$ Mild slope
 $\therefore y_n > y > y_c \Rightarrow M_2$ curve

$y_{\text{interval}} = \frac{y - y_c}{5} = \frac{3.186 - 2.17}{5} = 0.2$

$(y + \frac{q^2}{2gy^2}) (E_2 - E_1) \quad (\frac{qn}{y^{5/3}})^2 (S_{f1} + S_{f2})/2 \quad \frac{\Delta E}{(S_0 - S_f)}$

	$R_h = y \text{ (m)}$	$E \text{ (m)}$	$\Delta E \text{ (m)}$	S_f	$S_{\bar{f}}$	$\frac{\Delta E}{(S_0 - S_{\bar{f}})}$
1	2.17	3.25	0.03	5.11×10^{-3}		
2	2.37	3.28	0.06	3.81×10^{-3}	4.46×10^{-3}	-8.67
3	2.57	3.34	0.09	2.91×10^{-3}	3.36×10^{-3}	-25.42
4	2.77	3.43	0.12	2.26×10^{-3}	2.585×10^{-3}	-56.78
5	2.97	3.55	0.13	1.80×10^{-3}	2.03×10^{-3}	-116.50
6	3.17	3.68		1.44×10^{-3}	1.62×10^{-3}	-209.68
					$\Sigma L =$	-417 m

Exercise 2: water flowing at a normal depth in a rectangular concrete that is 12 m encounters an obstruction as show in figure below, causing the water level to rise above the normal depth at the obstruction and for some distance upstream. The discharge is $126 \text{ m}^3/\text{sec}$ and the channel bottom slope is 0.00086. The depth of water just upstream from the obstruction is 4.55 m. Find the distance upstream to the point where the water surface is at the normal depth.



Solⁿ:

$$B = 12 \text{ m} \quad Q = 126 \text{ m}^3/\text{s} \quad S_0 = 0.00086 \quad y_{\text{known}} = 4.55 \text{ m}$$

$$n = 0.013$$

$$Q = \frac{Q}{B} = \frac{126}{12} = 10.5 \text{ m}^2/\text{s} \Rightarrow y_c = \left(\frac{Q^2}{g} \right)^{1/3} = \left(\frac{10.5^2}{9.81} \right)^{1/3} = 2.24 \text{ m}$$

$$V = \frac{1}{n} R h S_0^{1/2} \quad Q = AV$$

$$Q = \frac{1}{n} A R h S_0^{1/2}$$

$$Q = \frac{1}{n} \frac{A^{5/3}}{P^{2/3}} S_0^{1/2}$$

$$126 = \frac{1}{0.013} \frac{(12 y_n)^{5/3}}{(12 + 2 y_n)^{2/3}} (0.00086)^{0.5} \quad \text{by trial and error}$$

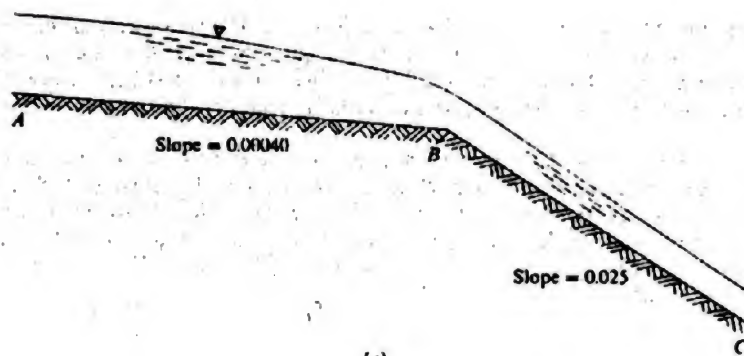
$$y_n = 2.95 \text{ m} \quad \therefore y_n > y_c \Rightarrow \text{Mild slope}$$

$$\therefore y > (y_n > y_c) \Rightarrow M1$$

$$y_{\text{interval}} = \frac{y_{\text{known}} - y_n}{5} = \frac{4.55 - 2.95}{5} = 0.32$$

$y \text{ (m)}$	$A \text{ (m}^2\text{)}$	$V \text{ (m/s)}$	$Rh \text{ (m)}$	$E \text{ (m)}$	$\Delta E \text{ (m)}$	S_f	\bar{S}_f	$dl \text{ (m)}$
4.55	54.6	2.31	2.59	4.82	-0.28	2.53×10^{-4}	2.815×10^{-4}	-484.4
4.23	50.76	2.48	2.48	4.54	-0.26	3.10×10^{-4}	3.48×10^{-4}	-507.8
3.91	46.92	2.69	2.37	4.28	-0.25	3.86×10^{-4}	4.38×10^{-4}	-592.4
3.59	43.08	2.92	2.25	4.03	-0.23	4.90×10^{-4}	5.65×10^{-4}	-779.7
3.27	39.24	3.21	2.12	3.80	-0.2	6.40×10^{-4}	7.50×10^{-4}	-1826.5
2.95	35.4	3.56	1.98	3.60		8.61×10^{-4}		$\Sigma L = -4190.4 \text{ m}$

Exercise 3: Water flows in a rectangular concrete channel that is 5 ft wide, as shown in figure below at a discharge of 16.5 ft³/sec. Find the water surface profile through the channel.



Handwritten calculations and diagrams for the water surface profile.

Given: $B = 5 \text{ ft}$, $Q = 16.5 \text{ ft}^3/\text{s}$, $n = 0.015$.

Channel slope: $S_1 = 0.0004$ (upstream), $S_2 = 0.025$ (downstream).

Flow regime: $y_n > y > y_c \Rightarrow M_2$ (upstream), $y_c > y > y_n \Rightarrow S_2$ (downstream).

Unit discharge: $q = \frac{Q}{B} = \frac{16.5}{5} = 3.3 \text{ ft}^2/\text{s}$

Critical depth: $y_c = \left(\frac{q^2}{g} \right)^{1/3} = \left(\frac{3.3^2}{32.2} \right)^{1/3} = 0.70 \text{ ft}$

Normal depth for S_1 : $Q = \frac{1.49}{n} A R^{2/3} S^{1/2}$

Normal depth for S_2 : $Q = \frac{1.49}{n} \frac{A^{5/3}}{P^{2/3}} S^{1/2}$

For S_1 : $16.5 = \frac{1.49}{0.015} (0.0004)^{1/2} \frac{(5 y_{n1})^{5/3}}{(5 + 2 y_{n1})^{2/3}} \Rightarrow y_{n1} = 1.66 \text{ ft}$

For S_2 : $16.5 = \frac{1.49}{0.015} (0.025)^{1/2} \frac{(5 y_{n2})^{5/3}}{(5 + 2 y_{n2})^{2/3}} \Rightarrow y_{n2} = 0.42 \text{ ft}$

For $\Delta L_1 = ?$ Take 3 interval $y_{int} = \frac{0.7 - 1.66}{3} = -0.32$

For $\Delta L_2 = ?$ Take 3 interval $y_{int} = \frac{0.42 - 0.7}{3} = -0.11$

$\Delta L_1 = ?$

$y \text{ (ft)}$	$A \text{ (ft}^2\text{)}$	$V \text{ (ft/s)}$	$R \text{ (ft)}$	$E \text{ (ft)}$	$\Delta E \text{ (ft)}$	S_f	$S_2 \text{ (ft)}$	ΔL_1
0.70	3.5	4.714	0.547	1.045	0.138	5.035 (10 ⁻³)	3.3335 (10 ⁻³)	-47.
1.02	5.1	3.235	0.724	1.183	0.251	1.632 (10 ⁻²)	1.1849 (10 ⁻²)	-313.8
1.34	6.7	2.463	0.872	1.434	0.287	7.378 (10 ⁻³)	5.697 (10 ⁻³)	-1691.2
1.66	8.3	1.988	0.998	1.721		4.016 (10 ⁻²)		
$\Sigma \Delta L_1 = -2058 \text{ ft}$								

$y \text{ (ft)}$	$A \text{ (ft}^2\text{)}$	$V \text{ (ft/s)}$	$R \text{ (ft)}$	$E \text{ (ft)}$	$\Delta E \text{ (ft)}$	S_f	S_2	ΔL_2
0.42	2.1	7.857	0.360	1.379	-0.223	0.024	0.0185	-34.3
0.513	2.565	6.433	0.426	1.156	-0.09	0.013	0.0104	-6.16
0.607	3.035	5.437	0.488	1.066	-0.021	7.737 (10 ⁻³)	6.416 (10 ⁻³)	-1.13
0.70	3.5	4.714	0.547	1.045		5.035 (10 ⁻³)		
$\Sigma \Delta L_2 = -41.6 \text{ ft}$								

The End